Unemployment Insurance and Optimal Taxation in a Search Model of the Labor Market*

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Abstract

In many search models of the labor market, unemployment insurance (UI) is conveniently interpreted as the value of leisure or home production and is, therefore, treated as a parameter. However, in reality, UI has to be funded through taxation that might be distortionary. In this paper, I analyze the welfare implications of raising funds towards UI benefits through different taxation systems within a directed search model. Since firms "direct" workers to apply to them by posting wages, raising UI funds in a lump-sum manner always distorts the efficient allocation, as it gives firms an incentive to be excessively aggressive in their attempt to maximize the probability of filing up their vacancies. I discuss two ways through which this externality can be internalized and efficiency can be restored.

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1 Introduction

In most search and matching models of the labor market, unemployment insurance (henceforth UI) is interpreted as the value of leisure or home production.[1] This assumption is convenient because it allows authors to treat UI as a parameter of the model. However, in reality, UI has to be funded through taxes, and these taxes may distort the economy's allocations and affect welfare. In this paper, I study a *directed search* model of the labor market, in which unemployment arises as an equilibrium result, and the government wishes to guarantee UI to all unemployed workers. I examine the optimal level of the unemployment benefit and the welfare implications of raising funds towards this benefit through various taxation systems.

In the standard directed search model, firms advertise their wage, and workers can direct their search towards specific wages. Equilibrium unemployment arises because firms have a limited number of vacancies, and workers cannot coordinate their application strategies. Hence, some firms can end up with a number of applications that exceeds their job openings, while others might receive no applications. The directed search model has become popular in the literature because it provides micro-foundations for the determination of the *matching function*, and it allows the study of *competition* among firms within markets that do not perfectly clear. In the core of this model, there is a trade-off between high wages and high matching probabilities. Firms who contemplate opening a vacancy and attracting workers realize this trade-off, thus internalizing the well-known search externality that governs the random search model (Hosios (1990)). Consequently, in the baseline directed search model, the equilibrium allocation is always efficient (for example, see Shimer (1996), Moen (1997), and Rogerson, Shimer, and Wright (2005)).

This paper focuses on the welfare implications of raising funds towards UI benefits under different taxation systems. I show the following result. Under lump-sum taxation, the very ingredient of the directed search model that delivers efficiency in the baseline case, namely *competition* for workers among firms, now generates an externality that distorts the economy and harms welfare. This channel works as follows. In a directed search framework, firms wish to advertise high wages because this guarantees a high probability of filling up their vacancies. Lump-sum taxation makes it cheap for firms to apply an aggressive wage policy because, under such taxation, they realize that their competitors will have to contribute equally to the unemployment that they caused by attracting more workers than they could employ. Of course, in *symmetric equilibrium* (studied in this paper), all firms advertise the same wage and receive the same expected number of applications. However, the aforementioned channel, creates an externality that leads to excessively high wages and inefficiently low entry of firms. The paper suggests two alternative systems of raising UI funds that deliver the efficient allocation.

Throughout this paper I consider a static directed search model of the labor market.[2] For simplicity, I assume that firms have only one vacancy, and workers can apply only to

[1] See for example Pissarides (2000).

[2] I focus on a one-shot game for simplicity. However, all the results of the paper can be easily generalized to a dynamic environment, if one focuses on steady states.

one firm.[3] Given that the authorities wish to guarantee a payment z to every unemployed worker, I consider two taxation systems. The first is a lump-sum system, according to which, every firm in the market pays an equal share of the UI bill (the product of z and the number of unemployed workers that arises in equilibrium). The second taxation system is personalized, and it dictates that each firm has to pay taxes that depend on how many applications it received. Since with directed search firms attract workers by posting public advertisements, I also consider a third alternative: I allow firms to advertise not only a wage paid to the employed worker, but also a payment made to workers who apply and do not obtain the job. This third system is based on an idea by Jacquet and Tan (2012), who refer to it as a "wage-vacancy" contract.

Since the focus of this paper is on the efficiency of funding unemployment benefits, rather than the provision of these benefits to workers, I first highlight the most important economic insights of the model in the more tractable case of risk-neutral workers. Under lump-sum taxation, the equilibrium wage is higher than in the model with no taxes, which might be surprising at first. Since in the model with UI firms expect that they will have to pay taxes, one might expect that they would at least promise lower wages. This reasoning turns out to be wrong. With lump-sum taxation firms have an incentive to post high wages in order to attract many workers, and they only have to pay for a small part of this aggressive behavior, since, at the end, all firms share equally the UI bill. Personalized taxes, that are Pigouvian in nature, induce firms to internalize this externality and lead to an equilibrium wage that is lower than in the model with no taxes.

After describing the wage and profits, I consider free-entry of firms. I determine the measure of active firms endogenously, and compare the decentralized equilibrium allocation with the Social Planners solution. It should be noted that, with risk-neutral workers, the Social Planner's objective is to choose the measure of active firms in order to maximize expected output, net of entry fees. In other words, insuring workers against the possibility of being unemployed does not improve welfare. However, this does not necessarily mean that the authorities might not wish to pay an UI to the unemployed. Perhaps the authorities want to achieve a more fair distribution of the output ex post, i.e. after the uncertainty regarding the workers' state has been resolved. This is especially true since, as I show, the authorities can promise *any* unemployment benefit to the unemployed without harming the economy's welfare.

Under lump-sum taxes, the typical firm's equilibrium profit is hump-shaped in the measure of active firms. Intuitively, a large measure of firms is bad for profits because it implies a low matching probability. At the same time, a low measure of firms is also bad for profits, because it implies high unemployment and few contributors to the UI bill. As a result, two equilibria exist: one with low entry, high per firm taxes, and a high matching probability, and one with high entry, low per firm taxes, and a low matching probability. Regardless of which equilibrium arises, the measure of active firms is

[3] Lester (2010), Tan (2012), Hawkins (2013), Geromichalos (2012), and Godenhielm and Kultti (2009) provide extensions of the directed search model, where firms can open more than one vacancies (or, alternatively, sellers can supply more than one units of a certain good). Also, Albrecht, Gautier, and Vroman (2006), Albrecht, Tan, Gautier, and Vroman (2004), and Galenianos and Kircher (2009) study different versions of the directed search model, where workers can apply to more than one firms.

always suboptimal. Hence, with lump-sum taxes, the authorities cannot pay benefits to the unemployed without hurting the economy's welfare. Personalized taxes resolve this inefficiency: the equilibrium profits under personalized taxes and under no taxes coincide. Hence, the equilibrium measure of firms is equal to the Social Planner's solution.[4] Under this type of taxation, the authorities can support *any level* of unemployment benefits without affecting the economy's efficiency. Interestingly, the equilibrium allocation is also optimal when the government does not use taxation, but rather requires the firms to post "wage-vacancy" contracts.

Finally, I consider the more empirically relevant case of risk-averse workers. In this case, the Planner's solution entails not only an efficient entry of firms, but also insuring the workers against the possibility of an unsuccessful search. I characterize the unique optimal level of UI, and I show that, under lump-sum taxation, this level can never be achieved as an equilibrium outcome. With personalized taxes, efficiency can be attained, but this requires that the authorities can set the UI level "just right", an assumption that might be questionable. I show that the most straightforward way to guarantee efficiency of the decentralized market allocation, is if the authorities do not choose the UI level (or the taxation system) themselves, but instead they simply require firms to post "wage-vacancy" contracts, *a la* Jacquet and Tan (2012).

This paper is closely related to the pioneering work of Acemoglu and Shimer (1999). In that paper, the authors consider a labor market with risk-averse workers and firms who make irreversible investments and post wages. As is standard in the directed search framework, workers realize that the probability of being offered a job at high-wage firms is low. UI encourages workers to seek jobs with higher wage (but also higher unemployment risk), and as a general equilibrium result firms respond to higher UI by creating high-wage, high-quality jobs, with greater unemployment risk. Hence, the authors show that moderate UI can not only increase welfare through risk sharing, but also by increasing output. The present paper highlights that these well-established positive effects of UI might be potentially jeopardized by a negative effect associated with UI payments (or, more precisely, with raising funds towards these payments). Hence, the authorities should not only be concerned about the existence of UI, but also about the way through which UI is funded. It is important to highlight that the externality identified in this paper arises in the case of lump-sum/non-personalized taxation, which is the system that most economies use in order to finance UI.

This paper is also closely linked to Golosov, Maziero, and Menzio (2012). Their goal is to study the optimal redistribution of income inequality in a directed search model with heterogeneous firms and moral hazard.[5] Julien, Kennes, King, and Mangin (2009) study the effects of public policy (including the provision of UI to the unemployed workers) within a directed search model of the labor market, and they contrast their findings to those that arise from a standard random matching model with Nash bargaining. Other papers that study optimal UI (not within a directed search framework) include Hansen

[4] Recall that with no taxes firm entry in the directed search model is always optimal.

[5] In Golosov, Maziero, and Menzio (2012) applying for a job is costly, and the authorities cannot distinguish between a worker who did not apply for a job and one who applied but was not offered one.

and İmrohoroğlu (1992), Hopenhayn and Nicolini (1997), Wang and Williamson (2002), Shimer and Werning (2008), and Rendahl (2012). The present paper is critically differentiated from the related literature, because it is, to the best of my knowledge, the only paper that studies the welfare implications of funding UI through different taxation systems. Hence, although many papers have focused on the provision of UI benefits to *workers*, this paper pays attention to how these benefits are funded (and the efficiency implications of the various alternatives), by focusing on the side of the *firms*.

The present paper is also closely related to Jacquet and Tan (2012), since one of the options that the authorities can adopt in order to raise UI funds is based on their "wage-vacancy" contracts. However, Jacquet and Tan (2012) do not study taxation. In my view, the present paper strengthens the results of Jacquet and Tan (2012), as it shows that their "wage-vacancy" contracts can do *at least as well as* a sophisticated government with personalized taxes in its disposal.

Finally, this paper is related to a number of works in the directed search literature that examine competition among firms (sellers) who can advertise general mechanisms rather than just a single wage (price). Examples of such papers include Julien, Kennes, and King (2000), Coles and Eeckhout (2003), Virág (2011), Eeckhout and Kircher (2010), and Geromichalos (2012). None of these papers study UI and/or taxes explicitly.

The rest of the paper is organized as follows. Section 2 provides a description of the physical environment and studies the Social Planner's allocation which will serve as a benchmark of comparison in order to assess the efficiency of the decentralized market equilibrium. Section 3 analyzes the case of risk-neutral workers and highlights the economic forces at work in the model. Then, Section 4 considers the more empirically relevant case of risk-averse workers. Finally, Section 5 concludes.

2 A Directed Search Model with UI and Taxes

I consider a static model of the labor market where firms post wages in order to attract workers. All the important results of the paper concern the so-called large market case, i.e. a market where there are continua of workers and firms. However, these results are obtained as the limiting cases of markets with a finite number of workers, n, and firms, m. Firms are risk-neutral. Workers' preferences admit an expected utility representation through the Bernoulli utility function $u(\cdot)$, with $u' > 0$ and $u'' \leq 0$. The case in which $u'' = 0$, so that workers are risk-neutral, is an interesting benchmark and the starting point of the analysis. Each firm has one vacancy. Any match within the period produces output y, so that matched firms make a profit equal to y net of the wage, w. Unmatched workers get an unemployment benefit z, and unmatched vacancies get nothing. The matching process and the wage advertisement of firms are determined through the directed search model described below. The size of z is chosen by the government in the beginning of the game, and I will consider two taxation systems for raising the necessary funds.

I now turn to the description of the matching process and the wage determination. I do not assume the existence of an aggregate matching function. Instead, I assume that workers rationally choose their application strategies in order to maximize expected utility. Workers can apply only to one firm, and if a firm receives multiple applications it offers the job to one of the applicants at random. Moreover, workers cannot coordinate their application strategies. Hence, some firms can end up with more than one application while others receive none. In this sense, the model gives rise to *equilibrium unemployment*.

Each firm posts a wage, $w \in [0, y]$, subject to promising a certain level of expected utility to the workers who apply for the job. In the main text, the typical firm takes this level of expected utility as given, because it is understood that the market is large enough so that a firm's own actions cannot affect market outcomes (recall that the focus of the paper is on the large market case). For completeness, in the accompanying Online Appendix I also describe equilibrium in the so-called small market, i.e. a market in which firms' actions can affect market outcomes.[6] Following the wage posting stage, workers observe all the wage advertisements and choose a probability of applying to each firm, taking as given the strategies of other workers. It is assumed that firms commit to their wage announcements, and I focus on wage announcements that lead to non-negative expected profit and expected utility for firms and workers, respectively.

Finally consider taxation. The government wants to guarantee a payment $z > 0$ to each unemployed worker, i.e. to each worker who remains unmatched after the matching game described above is played. The level of z is known to all market participants before the game starts, and I assume that $z < y$. The first way to raise funds, which I refer to as Taxation System A, is a lump-sum tax. In this setup, once the matching game has been completed, the number of unemployed workers is enumerated, and each firm has to pay an equal share of the UI bill.[7] For example, if $x < n$ workers are unmatched, each firm will pay a tax equal to xz/m. Alternatively, the government can adopt the personalized Taxation System B, under which, each firm pays z to each worker who applied to that specific firm but was not offered the position (because more than one workers applied).

In addition to Systems A and B, I also consider a third alternative. I study an extension of the baseline model where each firm can post a general schedule (w, z), where w is a payment (wage) made to the employed worker, and z is a payment (unemployment benefit) made to workers who applied to the firm in question but did not get a job. This specification follows Jacquet and Tan (2012), who refer to it as "wage-vacancy" contracts. For brevity, I refer to this environment as System C.

[6] Of course, since the focus of the paper is on the large market case, where $n, m \to \infty$, whether one models the strategic interaction among firms in the benchmark market with $m < \infty$ is immaterial: As $m \to \infty$, this interaction will vanish (for example, see Burdett, Shi, and Wright (2001)).

[7] Since in this paper it is the firms that make the interesting decisions, I assume that only firms pay taxes. Alternatively, one can assume that both firms and workers pay taxes, or even that only firms and employed workers pay taxes. These different assumptions would, of course, affect the equilibrium outcomes of the paper, but they would not change the spirit of the results. Hence, I adopt the assumption that makes the analysis as simple as possible. On the contrary, in Golosov, Maziero, and Menzio (2012) there are moral hazard considerations, and workers choose whether to apply to a firm or not. Hence, in their paper, taxation on workers becomes non-trivial. In fact, many of the interesting results of that paper concern income taxation.

2.1 Comparison of Equilibria with the Social Planner's Allocations

Letting $b \equiv n/m$ represent the market tightness, I normalize the measure of workers to the unit, as is standard in the literature, so that the term $1/b$ captures the measure of firms. Each firm is assumed to pay an entry fee $k > 0$ in order to enter the labor market, and the equilibrium measure of operating firms under System $i \in \{A, B, C\}$ is given by $1/b^i$, where I define

$$b^i \equiv \{b : \bar{\pi}^i(b) = 0\},$$

and $\bar{\pi}^i(b)$, $i \in \{A, B, C\}$, denotes the equilibrium profit in the large market (including the entry fee k that active firms have to pay).

The efficiency of equilibrium for various values of z, and under the various taxation systems, can be assessed by comparing the equilibrium b^i, $i \in \{A, B, C\}$, with the Social Planner's solution, which I now describe. With risk-neutral workers, there are no gains from risk sharing, and the Planner simply chooses the measure of firms that maximizes total output net of entry fees:

$$\max_b \left\{ \frac{1 - e^{-b}}{b} y - \frac{k}{b} \right\},$$

where $(1-e^{-b})/b$ is the probability with which the typical worker receives a job offer when the market tightness is given by b.[8] It can be easily verified that the unique maximizer of this problem is given by

$$b^* \equiv \{b : (1 - e^{-b} - be^{-b})y = k\}. \tag{1}$$

In the case of risk-averse workers, the efficiency of equilibrium under the various taxation systems depends, not only on the measure of active firms (as above), but also on whether the system can insure the risk-averse workers. Hence, with risk-averse workers, the choice of the amount z to be paid to unemployed workers becomes critical. The Social Planner is assumed to choose the level of z, as well as the (inverse of) the measure of active firms b, in order to maximize her objective function. The latter has to be defined in a way that facilitates reasonable comparison between the Planner's and the equilibrium

[8] As is standard in search theory (both directed and random), I assume that the Planner cannot bypass the search frictions that characterize the environment, i.e. she chooses b taking the matching probabilities as given. This assumption guarantees a "fair" comparison of the various equilibria with the Planner's allocation. Now, let Φ denote the probability with which the typical worker finds a job. In the text, I claimed that $\Phi = (1-e^{-b})/b$. To see why this is true, consider a finite market with n workers who apply to each of the m firms with equal probability, i.e. $1/m$ (as they will do in the symmetric equilibrium). For the typical firm j the probability of hiring *a worker* is $1-(1-1/m)^n$. Also, consistency requires that the probability with which the typical worker i obtains the job at firm j (conditional on applying) times the probability that worker i applies to firm j, i.e. $1/m$, should be equal to the probability that firm j offers the job to worker i, i.e. $[1-(1-1/m)^n]/n$. Mathematically this means that $\Phi/m = [1-(1-1/m)^n]/n$, or $\Phi = (m/n)[1-(1-1/m)^n]$. The term $(1-e^{-b})/b$ is simply the limit of the last expression as $n, m \to \infty$, keeping the market tightness $b \equiv n/m$ constant.

allocations. To that end, the key observation is that, due to free-entry, firms will make zero net profits in any equilibrium. Hence, the Planner's objective coincides with the workers' expected utility. Since all workers are identical and their measure is normalized to the unit, this objective is given by[9]

$$\max_{b,z}\left\{\frac{1-e^{-b}}{b}u\left(y+z-\frac{k+zb}{1-e^{-b}}\right)+\left(1-\frac{1-e^{-b}}{b}\right)u(z)\right\}.$$

For future reference define the objective function of the Planner (i.e. the term inside the curly bracket) as $\Omega(b,z)$. The term $(1-e^{-b})/b$ admits the same interpretation as above, and the term $y+z-(k+zb)/(1-e^{-b})$ denotes the wage paid to the worker under the zero-profit condition. In the appendix, I provide a more detailed discussion of the derivation of the Planner's objective function, and I show that the solution to her problem satisfies $b^S = b^*$, and $z^S = e^{-b^*}y$, which (not surprisingly) is the level of z that guarantees that workers are fully insured.

3 Risk-Neutral Workers

As I have already emphasized, the focus of this paper is on how firms (rather than workers) respond to the different incentives created by the various taxation systems that the authorities can adopt to raise UI funds. Therefore, although the case of risk-averse workers is more empirically relevant, some of the most important economic insights of the model can be best highlighted in the more tractable model with risk-neutral workers.

The starting point of the analysis is a market with a number of firms, m, which is finite but large enough, so that firms (correctly) perceive that their actions cannot affect market outcomes. Then, I obtain the limit as the market becomes infinitely large, keeping $b=n/m$ constant. Firms who wish to enter the labor market pay an entry fee $k>0$ in advance, and the equilibrium measure of firms is pinned down by a zero-profit condition. As I have already mentioned, I normalize the measure of workers to the unit, so that the market tightness b coincides with the inverse of the measure of firms. Let $\bar{w}^i(b),\bar{\pi}^i(b)$ denote the equilibrium wage and profit under System $i\in\{A,B,C\}$, and recall that the equilibrium market tightness satisfies $b^i=\{b:\bar{\pi}^i(b)=0\}$, $i\in\{A,B,C\}$.

The following proposition states the main results of this section. The derivation of these results is relegated to the appendix.

Proposition 1. *a) Under Systems A or B, the equilibrium wage is given by*

$$\begin{aligned}\bar{w}^A(b) &= z+\frac{be^{-b}}{1-e^{-b}}(y-z),\\ \bar{w}^B(b) &= z+b\frac{e^{-b}y-z}{1-e^{-b}},\end{aligned}$$

[9] Acemoglu and Shimer (1999) adopt a similar benchmark of efficiency (page 907, Section IV).

and for any $b > 0$ we have $\bar{w}^A(b) > \bar{w}^*(b) > \bar{w}^B(b)$, where $\bar{w}^*(b) \equiv be^{-b}y/(1-e^{-b})$, i.e. it denotes the equilibrium wage when $z = 0$.

Under System C, every pair $\left(w^C, z^C\right)$ that satisfies

$$(1-e^{-b})\bar{w}^C + (b-1+e^{-b})\bar{z}^C = be^{-b}y, \tag{2}$$

is consistent with equilibrium.

b) Under System $i \in \{A, B, C\}$, the equilibrium profit is given by

$$\begin{aligned}
\bar{\pi}^A(b) &= (1-e^{-b}-be^{-b})(y-z) - \left(1 - \frac{1-e^{-b}}{b}\right) bz - k, \\
\bar{\pi}^B(b) &= \bar{\pi}^C(b) = (1-e^{-b}-be^{-b})y - k \equiv \bar{\pi}^*(b),
\end{aligned}$$

and for any $b > 0$, we have $\bar{\pi}^A(b) < \bar{\pi}^*(b)$.

c) If the set $\mathcal{B}^A \equiv \{b : \bar{\pi}^A = 0\}$ is not empty, then it contains (generically) two elements, that is, equilibria under System A come in pairs. For any $b^A \in \mathcal{B}^A$, we have $b^A > b^*$, where b^* denotes the Planner's solution, defined in (1).

d) Under Systems B or C, $b^B = b^C = b^*$, hence, the (unique) equilibrium measure of firms coincides with the Social Planner's solution.

Proof. See the appendix. □

The comparison of equilibrium wages under Systems A,B delivers some interesting insights. Introducing UI and taxes in the model has a completely different effect on the equilibrium wage under these two systems. First, we have $w^B < w^* < w^A$ for all $b > 0$. Moreover, it is easy to verify that w^B is decreasing in z, while w^A is increasing. To better understand these results, notice that the introduction of UI and taxes creates several opposing forces, whose magnitude will ultimately determine whether $w^i < w^*$ or $w^i > w^*$. First, a higher z implies, on average, higher taxes for all firms. Second, a higher z increases the expected utility of workers so that any given firm can be attractive even if it does not post a very high wage. These two forces tend to put downward pressure on equilibrium wages, since they tend to weaken competition for workers among firms.

There is a third, less obvious force, which tends to put upward pressure on wages. To see how this force works, recall that a key feature of the directed search model is the trade-off between high wages and low probabilities of obtaining a job. In the model with no taxes, this mechanism puts a *constraint* on how aggressive firms want to be when posting wages, since firms realize that workers will not necessarily apply to them just because they offer a high wage (workers will be discouraged by the associated low probability of getting an offer).[10] Introducing unemployment benefits loosens the aforementioned *constraint*: when z is high, a firm that offers a high wage is extremely attractive because, even though workers understand that this firm will get many applications, the scenario of being unemployed does not look so bad to them anymore.

[10] This is the very reason why the so-called Bertrand Paradox does not arise in the standard directed search model. For more details, see Geromichalos (2014).

The direction of the three forces identified above is the same regardless of the taxation system. However, their magnitude critically depends on the choice of System A or B. Under System A, the third force is magnified because firms can increase wages and become very attractive at the expense of other firms who will equally share the UI bill. Put simply, under System A, firms post high wages in order to attract many workers, but they have to pay for a very small part of this aggressive behavior, since all firms will pay equally for the unemployment that a firm caused by offering a high wage and attracting workers whom it could not employ. Of course, in symmetric equilibrium, all firms post the same wage and receive the same expected number of applications. However, the externality described above leads to excessively high wages, so that $w^A > w^*$. On the other hand, under System B, firms realize that a high w will lead to very high taxes, and this minimizes the magnitude of the third force, so that, in equilibrium, $w^B < w^*$. Under System C, the equilibrium wage is indeterminate, but, as part (b) of the proposition reveals, the equilibrium profit is uniquely pinned down.[11]

Perhaps the most important result of Proposition 1 is that under Systems B and C the firms fully internalize the externality that they exert on one another under lump-sum taxation, so that, in equilibrium, $\bar{\pi}^B = \bar{\pi}^C = \bar{\pi}^*$, which also implies that $b^B = b^C = b^*$.[12] The key to this result is that, under either System B or C, firms realize that they cannot attract workers with extremely high wages, and expect other firms to pay for this aggressive behavior. Hence, if the government's objective is to achieve an equilibrium market tightness $b = b^*$, they have two ways to do it: either raise taxes through System B or require that firms should post "wage-vacancy" contracts, as in Jacquet and Tan (2012). However, notice that System B can implement *any* distribution of output between employed and unemployed workers (ex post), without affecting the economy's efficiency. This is not the case under System C. In fact, under System C, the distribution of output between employed and unemployed is indeterminate, since $\left(w^C, z^C\right)$ are indeterminate. This result will be altered in the case of risk-averse workers.

Figure 1 highlights that, under System A, the measure of active firms is not uniquely pinned down, because $\bar{\pi}^A$ is hump-shaped in the measure of firms (and, hence, it is also hump-shaped in b, which is the inverse of the measure of firms). The intuition is as follows. In the model with no taxes, a firm's equilibrium profit, $\bar{\pi}^*$, is always decreasing in the number of entrants (and increasing in b, as in the figure), since more firms make competition for workers harder. Raising funds through System A generates a new opposing force. Now too many firms (a low b) is bad for profits (for the same reason as above), but too few firms (a high b) is also bad, because it implies high unemployment and few contributors to the aggregate UI bill. As a result, two equilibria exist: one with many firms ($\bar{b}_1^A$ in the figure), low taxes and a low matching probability for firms, and

[11] This result reveals that the indeterminacy of the sharing rule that arises in models where firms can advertise general mechanisms rather than just a single wage, is only a feature of finite economies, i.e. environments where firms have market power. A more detailed discussion of this result can be found in the Online Appendix.

[12] This, in turn, implies that the authorities do not need to adopt complicated tax schemes (e.g. taxes proportional to wages or profits) in order to achieve efficiency, since both Systems B and C deliver the Planner's allocation.

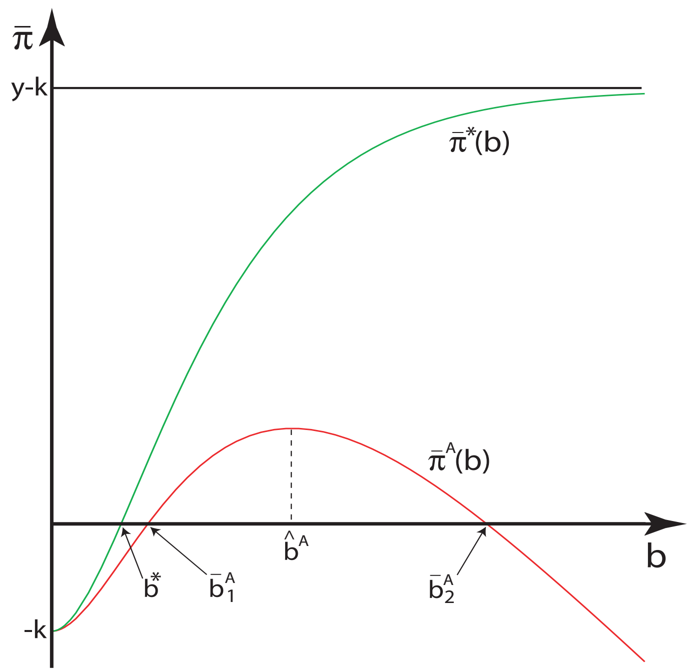

Figure 1: Determination of equilibrium b.

one with few firms ($\bar{b}_2^A$ in the figure), high taxes, but also a high matching probability. Importantly, the externality caused by the lump-sum taxation leads to a suboptimal entry of firms, regardless of which of the two equilibria will arise. Even though both equilibria are suboptimal, the one with high entry is relatively more efficient.[13]

4 Risk-Averse Workers

I now focus on the more empirically relevant case of risk-averse workers. Since this section contains some of the most important results of the paper, I describe equilibrium under the three possible systems in more detail (in three separate lemmas). Subsequently, I provide a comparison of the equilibrium welfare under the various systems (Proposition 2). Once again, the starting point of the analysis is a market with n workers, and a number of firms, m, which is finite, but large enough so that firms realize that their own actions cannot affect market outcomes. Then, I obtain the limit as the market gets infinitely large. Firms who wish to enter the labor market pay an entry fee $k > 0$ in advance, and the equilibrium measure of firms is pinned down by a zero-profit condition.

First, consider System A. Let θ represent the probability with which a typical worker

[13] This theory does not provide any claim that the more efficient of the two equilibria will arise.

applies to firm j. The profit maximization problem solved by firm j is:[14]

$$\max_{w^j} \left\{ [1-(1-\theta)^n](y-w^j) - \tilde{\tau}^A - k \right\}$$
$$s.t.\ \frac{1-(1-\theta)^n}{n\theta}u(w^j) + \left[1 - \frac{1-(1-\theta)^n}{n\theta}\right]u(z) = U.$$

The firm's objective is to maximize expected profit, subject to promising a certain level of expected utility, U, to the workers who apply for the job. The firm takes this level of expected utility as given, because a firm's own actions cannot affect market outcomes. Also, the firm takes the tax payment $\tilde{\tau}^A$ as given, because in a market with a large number of firms (and lump-sum taxation) whether firm j behaves more or less aggressively, will have an infinitesimal effect on the amount of taxes that it pays. Hence, this effect can be safely ignored.

The next lemma describes the symmetric equilibrium under Taxation System A.

Lemma 1. *Consider the model with risk-averse workers, and suppose that the government raises UI funds through System A. Define the function*

$$\begin{aligned} H^A(b) \quad &\equiv \quad (1-e^{-b}-be^{-b})\left[u\left(y+z-\frac{k+zb}{1-e^{-b}}\right)-u(z)\right] \\ &- \quad \frac{be^{-b}[(b-1+e^{-b})z+k]}{1-e^{-b}}u'\left(y+z-\frac{k+zb}{1-e^{-b}}\right). \end{aligned}$$

a) The equilibrium market tightness and wage are given by

$$\begin{aligned} \tilde{b}^A &= \{b>0 : H^A(b)=0\}, \\ \tilde{w}^A &= y+z-\frac{k+z\tilde{b}^A}{1-e^{-\tilde{b}^A}}. \end{aligned} \tag{3}$$

b) The function H^A is hump-shaped, hence, if equilibria exist, they come in pairs.

Proof. See the appendix. □

The equilibrium market tightness is implicitly given by (3). Since $H^A(b)$ is hump-shaped, if equilibria exist, they come (generically) in pairs. This multiplicity result is characteristic of System A regardless of the concavity of u. It stems from the fact that, under lump-sum taxation, a firm's profit is not monotonic in the market tightness. In the high $\tilde{b}^A$ equilibrium, few firms enter, so each firm has a high matching probability. On the downside, with few firms in the market, the number of unemployed workers is high, and each firm has to pay a big part of the UI bill. In the low $\tilde{b}^A$ equilibrium, the number of operating firms is large. This implies less unemployed workers and fewer taxes, but it

[14] The term $1-(1-\theta)^n$ captures the probability with which a firm receives one or more applications. The term $[1-(1-\theta)^n]/(n\theta)$ represents the probability with which a worker who applies to a firm which is chosen by other workers with probability θ gets the job. For details see footnote 17 in the appendix.

goes hand-in-hand with a lower matching probability. Of course, in both equilibria firms make zero profit, due to free-entry.

Next, consider the case in which the authorities adopt System B. The typical firm's maximization problem is:[15]

$$\begin{aligned}\max_{w^j} & \Big\{ [1-(1-\theta)^n]\,(y-w^j) - [n\theta - 1 + (1-\theta)^n]\,z - k \Big\} \\ s.t.\ & \frac{1-(1-\theta)^n}{n\theta}\,u(w^j) + \left[1 - \frac{1-(1-\theta)^n}{n\theta}\right] u(z) = U,\end{aligned}$$

The firm's wage posting strategy cannot affect the market utility, U. However, a firm that advertises a high wage is more attractive for workers, and under the personalized System B, it will have to pay taxes that are proportional to the number of applications it received. The next lemma describes the symmetric equilibrium under System B.

Lemma 2. *Consider the model with risk-averse workers, and suppose that the government raises UI funds through System B. Define the function*

$$\begin{aligned} H^B(b) &\equiv (1-e^{-b}-be^{-b})\left[u\left(y+z-\frac{k+zb}{1-e^{-b}}\right) - u(z)\right] \\ &- b\,\frac{e^{-b}k-(1-e^{-b}-be^{-b})z}{1-e^{-b}}\,u'\left(y+z-\frac{k+zb}{1-e^{-b}}\right).\end{aligned}$$

a) The equilibrium market tightness and wage are given by

$$\begin{aligned}\tilde{b}^B &= \{b>0 : H^B(b)=0\}, \\ \tilde{w}^B &= y+z-\frac{k+z\tilde{b}^B}{1-e^{-\tilde{b}^B}}.\end{aligned} \tag{4}$$

b) Under System B a unique equilibrium exists.

Proof. See the appendix. □

Despite its complexity, the function H^B is shown to be strictly increasing in b (see the appendix for some details on parameter restrictions), leading to the existence of a unique equilibrium, $\tilde{b}^B$. As in Section 3, System B leads to a unique equilibrium because it restores the monotonic (and positive) relationship between the average firm's profit and market tightness, b. Moreover, notice that under any parameter values,

$$H^B(b) - H^A(b) = b(1-e^{-b})u'\left(y+z-\frac{k+zb}{1-e^{-b}}\right)z > 0.$$

Therefore, for all $b>0$, we have $H^B(b) > H^A(b)$. This result is depicted in Figure 2, and it implies that $\tilde{b}^B < \tilde{b}^A$, regardless of which of the two equilibria arises under System A. Hence, as in Section 3, personalized taxes mitigate the aggressive wage posting behavior

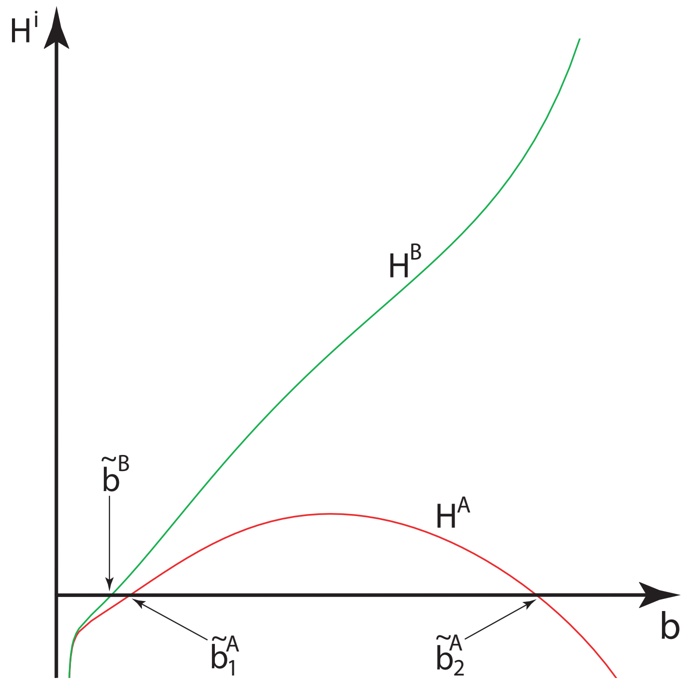

Figure 2: Equilibrium market tightness with risk-averse workers.

of firms and lead to a greater equilibrium measure of firms.

The last environment to be studied, is the one where firms post general contracts (w, z), i.e. System C. The typical firm j chooses a pair (w^j, z^j), taking as given the level of utility U that it must promise to its applicants. The typical firm's problem is:

$$\max_{w^j, z^j} \left\{ [1-(1-\theta)^n](y-w^j) - [n\theta - 1 + (1-\theta)^n] z^j \right\}$$
$$s.t. \ \frac{1-(1-\theta)^n}{n\theta} u(w^j) + \left[1 - \frac{1-(1-\theta)^n}{n\theta}\right] u(z^j) = U.$$

Clearly, the main difference between this problem and the firm's problem under Systems A,B, is that here z is not a parameter chosen by the government, but the firm's control variable. The following lemma characterizes the symmetric equilibrium under System C.

Lemma 3. *Consider the model with risk-averse workers, and suppose that firms post "wage-vacancy" contracts (i.e. System C is adopted). The equilibrium market tightness is given by $\tilde{b}^C = b^*$ (which is defined in (1)), and the equilibrium wage and UI satisfy*

$$\tilde{w}^C = \tilde{z}^C = e^{-b^*} y.$$

Proof. See the appendix. □

[15] The term $[n\theta - 1 + (1-\theta)^n]z$ represents the total tax payment for firm j. For details see footnote 20 in the appendix.

A very important and intuitive result that follows from Lemma 3, is that the risk-neutral firms will fully insure the risk-averse workers, by setting $\tilde{w}^C = \tilde{z}^C$. Moreover, unlike Section 3, here the equilibrium objects $\tilde{w}^C, \tilde{z}^C$ are uniquely determined. As Coles and Eeckhout (2003) point out, the indeterminacy of equilibrium in environments where firms post general mechanisms (as opposed to a single wage) stems from the fact that firms have many ways of responding optimally to their rivals' strategies (and keeping all other players, including workers, indifferent). In other words, each firm has a best response correspondence rather than a function. This is not the case when workers are risk-averse, since now a firm cannot keep workers indifferent by posting different levels of utility under different realizations of ex-post realized demand. Put simply, the firm's best response is a function rather than a correspondence. As a consequence, the objects $\tilde{w}^C, \tilde{z}^C$ are uniquely determined.

The last task of this section is to study the welfare properties of equilibrium under the different taxation systems and facilitate comparison. The relevant results are summarized in the following proposition. For this discussion it is important to recall the Social Planner's allocation for the economy with risk-averse workers (last part of Section 2.1).

Proposition 2. *a) Under System A, the Social Planner's allocation can never be achieved.*

b) Under System B, the Social Planner's allocation can be achieved if z is chosen optimally by the authorities. The optimal level of UI is given by $z = e^{-b^*} y$.

c) Under System C, the equilibrium allocation always coincides with the Social Planner's solution.

Proof. a) Clearly, $H^A(b) = 0$ cannot be satisfied at $b = b^*$ and $z = e^{-b^*} y$.

b) From Lemma 2, it can be easily verified that if $z = e^{-b^*} y$, then $\tilde{b}^B = b^*$ and $\tilde{w}^B = e^{-b^*} y$, is the unique equilibrium.

c) Follows immediately from comparison of Lemma 3 and the Planner's solution. □

In the model with risk aversion, efficiency requires not only a "correct" measure of firms, so that output net of entry costs is maximized, but also insuring the risk-averse workers. It turns out that the system that is guaranteed to deliver the efficient outcome is the one where the authorities do not even have to pick z themselves, i.e. System C. As long as firms have to post general wage-UI contracts, as in Jacquet and Tan (2012), the incentives of firms to direct workers to their job will lead to an equilibrium that coincides with the Planner's solution.

How well can the authorities do with taxes? System B can potentially deliver efficiency, but this requires that the authorities choose the level of z in a way that is "just right", in the sense that $z = e^{-b^*} y$. For any other z the equilibrium will be suboptimal, and whether the authorities are able to choose the optimal z with such precision is questionable. In any case, it is important to highlight that personalized taxes can potentially deliver the efficient allocation, if $z = e^{-b^*} y > 0$. This is important, since it is well-known that in a directed search model with risk-averse workers, the no-tax equilibrium does not coincide with the Planner's solution (for example, see Jacquet and Tan (2012)).

Finally, under System A efficiency can never be achieved, regardless of how wisely the authorities set z. With lump-sum taxation, the incentives of firms to direct workers to their jobs are always distorted, and an equilibrium with too few firms always prevails.

Proposition 1 states that efficiency can potentially be achieved under System B, and it can never be achieved under System A. However, the proposition does not claim that System B is superior to System A for any given z, as was the case with risk-neutral workers.[16] For any given z, it is true that $\tilde{b}^A > \tilde{b}^B$. Moreover, the probability with which a worker stays unemployed, $1 - (1 - e^{-b})/b$, is increasing in b. Hence, if z is relatively large, the second term of the welfare function Ω, defined in the Planner's problem, will tend to be larger under System A. In theory, this could imply that for some large z, $\Omega(\tilde{b}^A, z) > \Omega(\tilde{b}^B, z)$. However, recall that what makes System A inefficient is the distortion of incentives due to lump-sum taxes, and these taxes tend to be high when z is high. This second force is typically stronger. In a series of simulations, I have not been able to find parameter values that lead to $\Omega(\tilde{b}^A, z) > \Omega(\tilde{b}^B, z)$.

5 Conclusions

In this paper, I examine the welfare implications of raising funds towards unemployment insurance through different taxation systems. Lump-sum taxes distort the equilibrium outcomes and decrease welfare because they create an externality: Under lump-sum taxation, firms have an incentive to post high wages, in their attempt to increase the probability of filling up their vacancy, and doing so is rational because all firms share the unemployment benefit bill equally. In equilibrium, the measure of active firms is always suboptimal. One way to resolve this inefficiency is by introducing personalized taxes, i.e. a system which dictates that firms' tax payments will be proportional to the number of applications they receive. Another way to guarantee that the economy can achieve the Social Planner's allocation, in fact without imposing any taxes on firms, is to require firms to post general "wage-vacancy" contracts, *a la* Jacquet and Tan (2012).

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[16] In other words, the proposition states that if the authorities can choose z correctly, then System B is definitely better than System A, since it delivers the Planner's allocation. However, if the authorities choose a suboptimal z, it is not understood that System B will lead to greater efficiency than System A.

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A Appendix

The Social Planner's allocation in the model with risk-averse workers. First consider the Planner's objective function. As I have discussed in the main text, with firms making zero profits, the Planner wishes to maximize the typical worker's expected utility. Also, as I have shown in footnote 8, the term $(1 - e^{-b})/b$ is the probability with which the typical worker finds a job, and, hence, $1 - (1 - e^{-b})/b$ is the probability of being unemployed. Finally, the term $y + z - (k + zb)/(1 - e^{-b})$ is the wage paid to the worker under the zero profit condition for firms. To see this point more clearly, notice that in a market with worker-to-firm ratio equal to b, the expected profit of the typical firm is equal to $(1 - e^{-b})(y - w) - \tau - k$, where the term τ is determined by the balanced budget:

$$\left(1 - \frac{1 - e^{-b}}{b}\right) z = \frac{\tau}{b}.$$

Combining these observations, and solving the firm's zero profit condition with respect to the wage yields the desired result.

The first-order conditions with respect to b and z are necessary and sufficient for maximization of the Planner's problem. Notice that the first-order condition with respect to z reduces to

$$u'\left(y + z - \frac{k + zb}{1 - e^{-b}}\right) = u'(z),$$

which implies that the Planner's solution satisfies

$$k = \left(1 - e^{-b^S}\right) y - bz^S. \tag{a.1}$$

The first-order condition with respect to b can be written as

$$\begin{aligned}(1 - e^{-b} - be^{-b}) \left[u\left(y + z - \frac{k + zb}{1 - e^{-b}}\right) - u(z)\right] - \\ -b\,\frac{e^{-b}k - (1 - e^{-b} - be^{-b})z}{1 - e^{-b}}\,u'\left(y + z - \frac{k + zb}{1 - e^{-b}}\right) = 0.\end{aligned} \tag{a.2}$$

However, (a.1) implies that the term in square brackets in the first line of (a.2) is zero. Hence, (a.2) can only be satisfied if

$$e^{-b^S} k = \left(1 - e^{-b^S} - b^S e^{-b^S}\right) z^S. \tag{a.3}$$

It can now be easily verified that the unique solution to the system of equations defined by (a.1) and (a.3) is given by $b^S = b^*$ and $z^S = e^{-b^*} y$. □

Proof of Proposition 1. a) First, consider System A. Let θ represent the probability with which a typical worker applies to firm j. This firm's profit maximization problem is:[17]

$$\begin{aligned}&\max_{w^j} \left\{ [1 - (1 - \theta)^n]\,(y - w^j) - \tau^A - k \right\} \\ &s.t.\ \frac{1 - (1 - \theta)^n}{n\theta} w^j + \left[1 - \frac{1 - (1 - \theta)^n}{n\theta}\right] z = U.\end{aligned}$$

The firm's objective is to maximize expected profit, subject to promising a certain level of expected utility, U, to the workers who apply for the job. The firm takes this level of expected utility as given, because a firm's own actions cannot affect market outcomes.[18] Notice that the firm also takes the tax payment τ^A as given, because in a market with a

[17] The term $1 - (1 - \theta)^n$ represents the probability with which a firm receives one or more applications. Therefore, when a worker applies to a firm which is chosen by other workers with probability θ, the probability with which she will get the job is given by $[1 - (1 - \theta)^n]/(n\theta)$. This justifies the constraint in firm j's problem.

[18] In the literature, papers that adopt this assumption are said to follow the *market utility* approach. See for example Montgomery (1991) and Lang (1991).

large number of firms (and lump-sum taxation) the typical firm perceives that its behavior will not affect the amount of taxes that it will have to pay. Notice that the constraint in firm j's problem can be re-written as

$$[1-(1-\theta)^n]w^j = n\theta(U-z) + [1-(1-\theta)^n]z. \tag{a.4}$$

One can now replace the term $[1-(1-\theta)^n]w^j$ back into firm j's objective. This allows one to express the firm's maximization problem as a function of θ only. More precisely, firm j solves

$$\max_{\theta}\left\{[1-(1-\theta)^n](y-z) - n\theta(U-z) - \tau^A - k\right\}.$$

The first-order condition, which is necessary and sufficient, is given by

$$U - z = (1-\theta)^{n-1}(y-z).$$

Since the goal is to focus on symmetric equilibria, we can now impose the following conditions. First, all firms post the same wage, $w^j = w^A$. This, in turn, implies that all workers will apply to each firm with the same probability, $\theta = 1/m$. The latter condition implies that $U - z = (1-1/m)^{n-1}(y-z)$. Plugging this expression into the firm's indifference constraint (i.e. equation (a.4)), and solving with respect to $w^j = w^A$, yields the equilibrium wage under System A, which is as follows:[19]

$$w^A(n,m) = z + \frac{n}{m}\,\frac{(1-1/m)^{n-1}}{1-(1-1/m)^n}\,(y-z). \tag{a.5}$$

The last step (regarding System A) is to obtain the limiting expression $\bar{w}^A$ reported in Proposition 1. To that end, use the definition of the variable b, and re-write the equilibrium wage (equation (a.5)) as

$$w^A(n,m) = z + b\,\frac{(1-b/n)^{n-1}}{1-(1-b/n)^n}\,(y-z).$$

Since $\lim_{n\to\infty}(1-b/n)^n = e^{-b}$, it follows directly that

$$\bar{w}^A(b) = \lim_{n\to\infty} w_A(n,m) = z + \frac{be^{-b}}{1-e^{-b}}\,(y-z).$$

Next, consider the result concerning System B. Now the typical firm realizes that the amount of taxes it has to pay will be proportional to the number of applications it

[19] Clearly, the expression in (a.5) differs from the one reported in equation (1) in the Online Appendix, since the latter describes the equilibrium wage in a small market with strategic interaction. However, as one might expect, these expressions coincide "at the limit", i.e. as the market grows infinitely large. This result is intuitive, since the strategic interaction among firms vanishes as the market grows large.

received. Firm j's profit maximization problem is:[20]

$$\max_{w^j} \left\{ [1-(1-\theta)^n]\,(y-w^j) - [n\theta - 1 + (1-\theta)^n]\, z - k \right\}$$
$$s.t. \; \frac{1-(1-\theta)^n}{n\theta}\, w^j + \left[1 - \frac{1-(1-\theta)^n}{n\theta}\right] z = U.$$

The rest of the proof follows identical steps as the one for System A. First, one can exploit the indifference constraint of firm j in order to re-write the firm's maximization problem as a function of θ only. This problem is given by

$$\max_{\theta} \left\{ [1-(1-\theta)^n]y - n\theta U - k \right\}.$$

The first-order condition is given by

$$U = (1-\theta)^{n-1} y,$$

and imposing the symmetry conditions, $w^j = w^B$ and $\theta = 1/m$, yields[21]

$$w^B(n,m) = z + \frac{n}{m} \frac{(1-1/m)^{n-1}\, y - z}{1-(1-1/m)^n}. \tag{a.6}$$

The derivation of the limiting expression $\bar{w}^B$, reported in Proposition 1, involves the same steps as the ones we followed to derive $\bar{w}^A$ earlier. The details are therefore omitted. Also, showing that $\bar{w}^A(b) > \bar{w}^*(b) > \bar{w}^B(b)$ is trivial.

Finally, consider System C. Firm j's profit maximization problem is:

$$\max_{w^j, z^j} \left\{ [1-(1-\theta)^n]\,(y-w^j) - [n\theta - 1 + (1-\theta)^n]\, z^j - k \right\}$$
$$s.t. \; \frac{1-(1-\theta)^n}{n\theta}\, w^j + \left[1 - \frac{1-(1-\theta)^n}{n\theta}\right] z^j = U.$$

This problem is mathematically identical to the one that the firm faces under System B, with the exception that the policy parameter z has now been replaced with the term z^j, which is a control variable for the firm. Since the two problems are mathematically identical, one can still write the firm's problem as a function of θ only, and the first-order condition $U = (1-\theta)^{n-1}y$ is still necessary and sufficient for profit maximization. Since we have a unique condition that characterizes the typical firm's optimal behavior, while, in reality, the firm is choosing both w^j and z^j, it is not surprising that the equilibrium is characterized by an indeterminacy. In particular, any pair (w^C, z^C) that satisfies

$$\left[1 - \left(1 - \frac{1}{m}\right)^n\right] w^C = \left[1 - \left(1 - \frac{1}{m}\right)^n - \frac{n}{m}\right] z^C + \frac{n}{m}\left(1 - \frac{1}{m}\right)^{n-1} y, \tag{a.7}$$

[20] When workers apply to a firm with probability θ, the number of workers to whom that firm cannot offer a job is $\sum_{i=1}^{n} \binom{n}{i} (1-\theta)^{n-i}\, \theta^i (i-1)$, and one can verify that it is equal to $n\theta - 1 + (1-\theta)^n$. Hence the term $[n\theta - 1 + (1-\theta)^n]z$ inside firm j's profit represents the total tax payment for that firm.

[21] Regarding the relationship between the equilibrium wage in (a.6) and the one reported in equation (2) in the Online Appendix, a comment similar to the one in footnote 19 also applies here.

is consistent with equilibrium. Equation (2) in the proposition can be obtained by replacing $1/m$ with b/n in equation (a.7), and taking the limit as $n \to \infty$.

b) The profit expressions reported in the proposition can be derived fairly easily, given the analysis in part (a). For instance, consider System A. The symmetric equilibrium profit of the typical firm is given by

$$\pi^A(n,m) = \left[1 - \left(1 - \frac{1}{m}\right)^n\right](y - w^A) - \tau^A - k.$$

In the symmetric equilibrium, the probability with which the typical worker finds a job is given by $(m/n)[1-(1-1/m)^n]$ (see footnote 17 for details). Hence, the probability with which she does not find a job is $1-(m/n)[1-(1-1/m)^n] \equiv p_u$, and the expected amount of taxes that the typical firm has to pay equals $\tau^A = p_u nz/m$, or

$$\tau^A = \left[\frac{n}{m} - 1 + \left(1 - \frac{1}{m}\right)^n\right] z.$$

Therefore, one can re-write the typical firm's profit as

$$\pi^A(n,m) = \left[1 - \left(1 - \frac{1}{m}\right)^n\right](y - w^A) - \left[\frac{n}{m} - 1 + \left(1 - \frac{1}{m}\right)^n\right] z - k.$$

As the market grows large, we have $w^A \to \bar{w}^A$, and after some algebra one can verify that

$$\bar{\pi}^A(b) = (1 - e^{-b} - be^{-b})(y - z) - \left(1 - \frac{1 - e^{-b}}{b}\right) bz - k.$$

The derivations for $\bar{\pi}^B(b), \bar{\pi}^C(b)$ follow identical steps, hence, they are omitted.

The last statement in part (b) of the proposition claims that $\bar{\pi}^A(b) < \bar{\pi}^*(b)$, where $\bar{\pi}^*(b) \equiv (1 - e^{-b} - be^{-b})y - k$. It is straightforward to verify that this inequality will be satisfied if and only if $b(1-e^{-b})z > 0$, which is true for all $b, z > 0$.

c) For convenience define $F(b) \equiv \bar{\pi}^A(b) + k$, so that the equilibrium market tightness under System A is given by the solution to the equation $F(b) = k$. Notice that $F(0) = 0$ and $F'(b) = be^{-b}(y - z) - (1 - e^{-b})z$. Hence, for any $y \le 2z$

$$F'(b) \le 2be^{-b}z - be^{-b}z - (1 - e^{-b})z = -z(1 - e^{-b} - be^{-b}),$$

which is strictly negative for all $b, z > 0$. Therefore if $z \ge y/2$, the profit is never positive, and no firms will ever enter in the market, no matter how small k might be. From now on, I assume that $z < y/2$.

Assuming that $z < y/2$, implies that there exist b small enough, such that $F'(b) > 0$. Moreover, it is straightforward to see that $\lim_{b\to\infty} F(b) = -\infty$. Therefore, if an equilibrium exists, it will not be generically unique (recall that $F(0) = 0$). But the proposition claims something stronger: if equilibria exist, they always comes in pairs. This will be true if the function $F(b)$ has a unique interior maximum. I will show that this is the case, by proving that the set $\mathcal{B}'_0 \equiv \{b > 0 : F'(b) = 0\}$ is a singleton (I have already shown that it is non-empty). For future reference, notice that $F'(0) = 0$ and

$$F''(b) = e^{-b}(y - 2z) - be^{-b}(y - z). \tag{a.8}$$

Assume, by way of contradiction, that $\mathcal{B}'_0$ is not a singleton, and define $b_1 \equiv \min\{\mathcal{B}'_0\}$. Notice that since for $b \approx 0$, $F'(b) > 0$, it must be that $F''(b_1) < 0$. Under the contradictory assumption, there exists $b_2 \in \mathcal{B}'_0$, such that $b_2 > b_1$ and $F''(b_2) > 0$. Moreover, since $b_1, b_2 \in \mathcal{B}'_0$, we have

$$b_1 e^{-b_1}(y - z) = (1 - e^{-b_1})z, \tag{a.9}$$

$$b_2 e^{-b_2}(y - z) = (1 - e^{-b_2})z. \tag{a.10}$$

Combining (a.8) with (a.9) and (a.10), one can write

$$F''(b_1) = e^{-b_1}(y - z) - z,$$

$$F''(b_2) = e^{-b_2}(y - z) - z.$$

Since $F''(b_1) < 0$ and $F''(b_2) > 0$, one can conclude that

$$z - e^{-b_1}(y - z) > 0, \tag{a.11}$$

$$e^{-b_2}(y - z) - z > 0. \tag{a.12}$$

Adding up (a.11) and (a.12) implies that

$$\left(e^{-b_2} - e^{-b_1}\right)(y - z) > 0,$$

which is a contradiction, since $y > z$ and $b_2 > b_1$.

Based on the discussion so far one can conclude that if equilibria (i.e. solutions to $F(b) = k$) exist, they will always come in pairs. It is easy to verify that, under System A, equilibria with a positive measure of firms will exist if and only if

$$F(\hat{b}^A) = \left(1 - e^{-\hat{b}^A}\right)(y - z) - \hat{b}^A z \geq k \quad \Leftrightarrow \quad \bar{\pi}^A(\hat{b}^A) \geq 0,$$

where $\hat{b}^A \equiv \{b : be^{-b}(y - z) = (1 - e^{-b})z\}$ is indicated in Figure 1.

Finally, the claim that $b^A > b^*$, for all $b^A \in \mathcal{B}^A$, follows directly from the fact that $\bar{\pi}^A(b) < \bar{\pi}^*(b)$, for all $b > 0$, which was shown in part (b).

d) This part is trivial. □

Proof of Lemma 1. a) Consider the problem of the typical firm j in the market with very large (yet finite) n, m. Although the maximization problem of the firm is very similar to the one solved in part (a) of Proposition 1, here u is not linear, hence, we cannot follow the same method as earlier (namely, solve the indifference constraint with respect to the wage, and plug it into the firm's objective in order to express the latter as a function of θ

only). Instead, we will exploit the Implicit Function Theorem (IFT). The necessary and sufficient first-order condition (with respect to w^j) for profit maximization satisfies

$$-[1-(1-\theta)^n]+n(1-\theta)^{n-1}(y-w^j)\frac{\partial\theta}{\partial w^j}=0.$$

Applying the IFT in the expected utility constraint of the firm, we can obtain an expression for $\partial\theta/\partial w^j$. In particular, one can show that

$$\frac{\partial\theta}{\partial w^j}=-\frac{1}{n}\frac{[1-(1-\theta)^n]\,u'(w^j)}{(1-\theta)^{n-1}[u(w^j)-u(z)]+u(z)-U}. \tag{a.13}$$

Substituting (a.13) into the first-order condition, and obtaining the term U from the expected utility constraint, implies that the typical firm chooses a w^j that satisfies

$$\left[\frac{1-(1-\theta)^n}{n\theta}-(1-\theta)^{n-1}\right]\left[u(w^j)-u(z)\right]-(1-\theta)^{n-1}(y-w^j)u'(w^j)=0. \tag{a.14}$$

At this point, some important observations are in order. First, since the focus is on symmetric equilibria, it must be $w^j=w$. Moreover, under symmetry, the tax that each firm will have to pay is given by $\tilde{\tau}^A=[n/m-1+(1-1/m)^n]z$. Finally, in equilibrium all firms make zero profit, which implies that

$$y-w=\frac{\tilde{\tau}^A+k}{1-(1-1/m)^n}=\frac{[n/m-1+(1-1/m)^n]\,z+k}{1-(1-1/m)^n}.$$

Incorporating these observations in (a.14), substituting $1/m=b/n$, and taking the limit as $n\to\infty$, yields the equilibrium condition given in (3).

b) The proof is similar to the one of part (c) of Proposition 1. A sufficient condition for existence of equilibrium is $\max_b H^A(b)\geq 0$. In part (c) of Prop. 1, it was possible to obtain a closed-form solution for the analogous condition, but this is not the case here. □

Proof of Lemma 2. a) Taking the first-order condition in firm j's profit with respect to w^j implies

$$-[1-(1-\theta)^n]+n(1-\theta)^{n-1}(y-w^j)\frac{\partial\theta}{\partial w^j}-[n-n(1-\theta)^{n-1}]z\frac{\partial\theta}{\partial w^j}=0.$$

Since the expected utility constraint in the firm's problem is identical to the one under System A, $\partial\theta/\partial w^j$ is still given by (a.13). Substituting this expression back into the first-order condition, and obtaining the term U from the expected utility constraint, implies that the typical firm chooses a w^j that satisfies

$$\left[\frac{1-(1-\theta)^n}{n\theta}-(1-\theta)^{n-1}\right]\left[u(w^j)-u(z)\right]-[(1-\theta)^{n-1}(y+z-w^j)-z]u'(w^j)=0. \tag{a.15}$$

The rest of the proof proceeds in the same steps as the one of Lemma 1. First, since the focus is on symmetric equilibria, $w^j=w$. Moreover, under symmetry, each firm's tax bill is given by $\tilde{\tau}^B = [n/m - 1 + (1-1/m)^n]z$. Finally, in equilibrium all firms make zero profit, which implies that

$$y - w = \frac{[n/m - 1 + (1-1/m)^n]\, z + k}{1-(1-1/m)^n}.$$

Incorporating these observations in (a.15), substituting $1/m = b/n$, and taking the limit as $n \to \infty$, yields the equilibrium condition given in (4).

b) Define the terms

$$f(b) \equiv 1 - e^{-b} - be^{-b},$$
$$g(b) \equiv \frac{k+zb}{1-e^{-b}}.$$

Using these definitions, H^B can be re-written as

$$H^B(b) = f(b)\left\{u\left[y+z-g(b)\right] - u(z)\right\} - b[e^{-b}g(b) - z]u'\left(y+z-\frac{k+zb}{1-e^{-b}}\right).$$

Notice that $f(b) \in (0,1)$, for all $b > 0$, with $f'(b) = be^{-b} > 0$, for all $b > 0$.

The function $g(b)$ is key for this proof, so I discuss it in detail. It satisfies $g(b) > 0$, for all $b > 0$, and also $\lim_{b\to 0} g(b) = \infty = \lim_{b\to\infty}$. Moreover,

$$g'(b) = \frac{f(b)z - ke^{-b}}{(1-e^{-b})^2} = -\frac{g(b)e^{-b} - z}{1-e^{-b}}. \tag{a.16}$$

Hence, $g(b)$ is u-shaped, and it attains a unique global minimum, at $b = b^m \equiv \{b : f(b)z = ke^{-b}\}$. From now on, I will assume that the following condition holds:

$$g(b^m) = \frac{z}{e^{b^m}} < y. \tag{a.17}$$

In what follows, I show that this is a sufficient condition for existence and uniqueness of equilibrium.[22] Notice that if (a.17) holds true, there exists a range of b's for which $g(b) < y$. Hence, (a.17) is a fairly weak requirement on how big k, z can be: it guarantees that, given the announcement z (and the cost k), there will exist equilibria where firms post a wage which is no less than z. Assuming that (a.17) holds, there exist exactly two values of $b > 0$, for which $g(b) = y$. Let me define them as b_1^y, b_2^y, with $b_1^y < b_2^y$.

I now prove that, for all $b \in [b_1^y, b_2^y]$, H^B is strictly increasing.

$$\begin{aligned}\frac{dH^B(b)}{db} &= f'(b)\left\{u\left[y+z-g(b)\right] - u(z)\right\} - f(b)\, g'(b)\, u'\left[y+z-g(b)\right] \\ &- [e^{-b}g(b) - z]u'\left[y+z-g(b)\right] - be^{-b}\left[g(b) - g'(b)\right]u'\left[y+z-g(b)\right] \\ &+ b[e^{-b}g(b) - z]\, g'(b)\, u''\left[y+z-g(b)\right].\end{aligned}$$

[22] This requirement is not necessary for existence of equilibrium. It can be shown that equilibrium exists even if $g(b^m) \geq y$.

Notice that this derivative is the sum of the following five terms:

$$
\begin{aligned}
h_1(b) &= f'(b)\left\{u\left[y+z-g(b)\right]-u(z)\right\},\\
h_2(b) &= -\,f(b)\,g'(b)\,u'\left[y+z-g(b)\right],\\
h_3(b) &= -\,[e^{-b}g(b)-z]u'\left[y+z-g(b)\right],\\
h_4(b) &= -\,be^{-b}\left[g(b)-g'(b)\right]u'\left[y+z-g(b)\right],\\
h_5(b) &= b[e^{-b}g(b)-z]\,g'(b)\,u''\left[y+z-g(b)\right].
\end{aligned}
$$

Out of these terms, only h_1 is undoubtedly positive. I claim that h_5 is also positive. To see why this is true, substitute the term $g'(b)$ from (a.16) into h_5 to obtain

$$h_5(b) = -\frac{b}{1-e^{-b}}[e^{-b}g(b)-z]^2\,u''\left[y+z-g(b)\right],$$

which is strictly positive for all $b>0$, since $u''(\cdot)<0$.

The proof of monotonicity of H^B will be complete, if it can be shown that $h_2(b)+h_3(b)+h_4(b)\geq 0$. Before examining this term, notice that

$$g(b)-g'(b) = \frac{g(b)-z}{1-e^{-b}}.$$

Using this fact, one can re-write the fourth term as

$$h_4(b) = be^{-b}\,\frac{g(b)-z}{1-e^{-b}}\,u'\left[y+z-g(b)\right].$$

Hence, the requirement that $h_2(b)+h_3(b)+h_4(b)\geq 0$ will be true if and only if

$$u'\left[y+z-g(b)\right]\left\{-f(b)g'(b)-[e^{-b}g(b)-z]+be^{-b}\,\frac{g(b)-z}{1-e^{-b}}\right\}\geq 0.$$

Substituting for $g'(b)$ from (a.16) and rearranging terms, implies that the latter requirement will be true if and only if

$$be^{-b}g(b)\geq 0.$$

Of course, this condition holds (with strict inequality) for all $b>0$. Hence, $dH^B/db>0$.

It follows that if an equilibrium $b\in[b_1^y,b_2^y]$ exists, it will be unique. To obtain existence, notice that

$$
\begin{aligned}
H^B(b_1^y) &= -\,b_1^y\left[e^{-b_1^y}g(b_1^y)-z\right]u'(z),\\
H^B(b_2^y) &= -\,b_2^y\left[e^{-b_2^y}g(b_2^y)-z\right]u'(z).
\end{aligned}
$$

Luckily, the term $e^{-b}g(b)-z$ also appears in $g'(b)$ in (a.16), and the sign of the expressions $g'(b_1^y)$ and $g'(b_2^y)$ is known (by the very definition of b_1^y, b_2^y). We have

$$
\begin{aligned}
\text{sign}\{H^B(b_1^y)\} &= \text{sign}\left\{-\left[e^{-b_1^y}g(b_1^y)-z\right]\right\} = \text{sign}\{g'(b_1^y)\} = -,\\
\text{sign}\{H^B(b_2^y)\} &= \text{sign}\left\{-\left[e^{-b_2^y}g(b_2^y)-z\right]\right\} = \text{sign}\{g'(b_2^y)\} = +.
\end{aligned}
$$

Summing up, the function $H^B(b)$ is strictly increasing and continuous in $b\in[b_1^y,b_2^y]$, with $H^B(b_1^y)<0$, and $H^B(b_2^y)>0$. Hence, a unique equilibrium always exists. □

Proof of Lemma 3. The first-order conditions with respect to w^j and z^j, respectively, are

$$-[1-(1-\theta)^n]+n(1-\theta)^{n-1}(y-w^j)\frac{\partial\theta}{\partial w^j}-[n-n(1-\theta)^{n-1}]z\frac{\partial\theta}{\partial w^j}=0,$$
$$-[n\theta-1+(1-\theta)^n]+n(1-\theta)^{n-1}(y-w^j)\frac{\partial\theta}{\partial z^j}-[n-n(1-\theta)^{n-1}]z\frac{\partial\theta}{\partial z^j}=0.$$

Applying the IFT on the expected utility constraint in the firm's problem, one can show that

$$\frac{\partial\theta}{\partial w^j}=-\frac{1}{n}\frac{[1-(1-\theta)^n]\,u'(w^j)}{(1-\theta)^{n-1}[u(w^j)-u(z^j)]+u(z^j)-U},$$
$$\frac{\partial\theta}{\partial z^j}=-\frac{1}{n}\frac{[n\theta-1+(1-\theta)^n]\;u'(z^j)}{(1-\theta)^{n-1}[u(w^j)-u(z^j)]+u(z^j)-U}.$$

Substituting these expressions into the first-order conditions yields

$$(1-\theta)^{n-1}[u(w^j)-u(z^j)]+u(z^j)-U=-u'(w^j)[(1-\theta)^{n-1}(y+z^j-w^j)-z^j], \quad \text{(a.18)}$$
$$(1-\theta)^{n-1}[u(w^j)-u(z^j)]+u(z^j)-U=-u'(z^j)[(1-\theta)^{n-1}(y+z^j-w^j)-z^j]. \quad \text{(a.19)}$$

Equations (a.18) and (a.19) reveal that the firm will fully insure the risk-averse workers, by setting $w^j=z^j$. To characterize the equilibrium values, impose $w^j=z^j$ in either (a.18) or (a.19), together with symmetry. This implies that the equilibrium value of w in the finite market solves

$$u'(w)\left[\left(1-\frac{1}{m}\right)^{n-1}y-w\right]=0\;\Rightarrow\; w=\left(1-\frac{1}{m}\right)^{n-1}y.$$

To obtain an expression for the large market, substitute $1/m$ with b/n in the expression above, and take the limit as $n\to\infty$. We have

$$\tilde{w}^C=\tilde{z}^C=e^{-b}y.$$

The last step is to characterize the equilibrium level of b. The equation that pins down this object is the zero-profit condition. In the large market, under symmetry, the profit for each firm is given by

$$(1-e^{-b})(y-\tilde{w}^C)-(b-1+e^{-b})\tilde{z}^C-k.$$

Equating this expression with zero, and using the fact that $\tilde{w}^C=\tilde{z}^C$, implies that the equilibrium market tightness solves $(1-e^{-b}-be^{-b})y=k$, i.e. $\tilde{b}^C=b^*$. □