Over-the-Counter Trade and the Value of Assets as Collateral

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ABSTRACT

We study asset pricing within a general equilibrium model where unsecured credit is ruled out, and a real asset helps agents carry out mutually beneficial transactions by serving as collateral. A unique feature of our model is that the agent who provides the loan might have a low valuation for the collateral asset. Nevertheless, the lender rationally chooses to accept the collateral because she can access a secondary asset market where she can sell the asset. Following a recent strand of the finance literature, based on the influential work of Duffie, Gârleanu, and Pedersen (2005), we model this secondary asset market as an over-the-counter market characterized by search and bargaining frictions. We study how the asset's property to serve as collateral affects its equilibrium price, and how the asset price and the economy's welfare are affected by the degree of liquidity in the secondary asset market.

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1 Introduction

What makes an automobile, a house, or a T-bill a better form of collateral than a piano or a piece of artwork? Clearly, the desirability of the asset by the lender is not a necessary condition for its use as collateral. For instance, when a bank provides a loan to an individual and accepts her house as collateral, it is not because the bank plans to use (i.e., keep and obtain utility from) the house in the event of default. In this example, two are the key features that constitute the house a good form of collateral. First, the borrower (and not necessarily the lender) does have a high valuation for the house, and this ensures that she has an incentive to honor her debt and not lose the house. Second, even if the borrower defaults, the bank can access a well-organized secondary market where it can sell the collateral. Hence, objects such as a piano or a piece of artwork may serve poorly as a collateral not because lenders do not value them (this may well also be the case for the automobile or the house), but because they fear that it will be relatively hard to sell them due to the lack of a *liquid* secondary market for these objects.

To formalize these ideas, we develop a model in which part of the economic activity takes place in markets where certain frictions, such as anonymity and limited commitment, obstruct unsecured credit. With imperfect credit a role for a medium of exchange and/or collateral arises naturally. Agents who wish to purchase goods in the frictional market can use (fiat) money as a means of payment. In addition to money, there exists a second asset which the agents can pledge as a collateral in order to obtain a secured loan and increase their consumption, if they find themselves short of cash. In line with the earlier discussion, lenders have a potentially lower valuation for this asset.[1] Nevertheless, the lenders who have offered loans to borrowers who renege on their debts can visit a secondary asset market and sell the collateral. To provide a precise concept of asset market liquidity we follow the recent, influential work of Duffie, Gârleanu, and Pedersen (2005), and assume that the secondary asset trade takes place in over-the-counter (OTC) markets characterized by search and bargaining frictions.

Within this framework, we are able to address a number of intersting questions, such as: How does the ability of the asset to serve as collateral affect its equilibrium price? How does the asset price and the economy's welfare depend on the liquidity of the secondary asset market (i.e., on how easy it is for lenders-owners of collateral to find buyers for these assets)? Last but not least, can monetary policy affect equilibrium asset prices and welfare, and how?

We find that the asset's property to serve as a collateral critically affects its equilibrium price. As long as the supply of the asset is not very plentiful, its price will always include a *liquidity premium*, i.e., it will exceed the price that the asset would obtain in an environment where agents

[1] The argument that a lender might have a "low valuation" for an asset such as a car or a house, as in the earlier example, is self-explanatory. When it comes to financial assets, such as T-bills, the notion of a low asset valuation is less straightforward, since these assets typically pay a predetermined cash flow. One way to motivate the concept of low valuation for a financial asset is to think that this asset has a long maturity horizon (i.e., it pays in the future), while the lender might have an immediate liquidity need.

hold it purely for its role as a store of value. This liquidity premium stems from the fact that an additional unit of asset can help the agent obtain a greater loan and increase her consumption. In recent work, Krishnamurthy and Vissing-Jorgensen (2012) report that assets, such as T-bills, enjoy significant liquidity premia. Our paper can help rationalize this empirical regularity.

Since agents can purchase more goods by using either money (in a "quid pro quo" fashion) or the asset (by pledging it as a collateral), these two objects are effectively substitutes. An increase in inflation, which can be thought of as an increase in the cost of holding money, makes the real asset relatively cheaper inducing agents to increase their demand for the asset and, consequently, its price. Hence, developing a model where the asset is valued for its property to facilitate transactions by serving as collateral, might be key for understanding the negative (positive) relationship between asset returns (prices) and inflation, which is well-documented in the empirical finance literature (for instance, see Marshall (1992)).

Our paper provides a framework within which we can study the effect of the secondary asset market liquidity on asset prices. We show that a higher probability of trade in the OTC market increases the price of the asset, even if the original buyers of the asset, i.e., the agents who plan to use it as a collateral in order to issue a secured loan in the goods market, do not directly participate in the OTC asset market. This is true because a higher liquidity in the OTC market makes lenders more willing to accept the collateral, since they expect that it will be relatively easy to sell it off in the case that the borrower defaults. We also show that the asset price is increasing in the bargaining power of the lender in the OTC market (i.e., the seller of assets in that market). Interestingly, lenders (rationally) choose to provide loans and accept the asset as collateral, even in the extreme case in which their personal valuation for the asset is zero.

The result that the asset price is increasing in the liquidity of the secondary asset market is not only intuitive, but also consistent with anecdotal evidence. For instance, the Assistant Secretary of the US Treasury clearly implies that secondary market liquidity is important because it encourages "more aggressive bidding in the primary market", that is, it leads to a higher issue price, thus, allowing the Treasury to borrow funds at a cheaper rate.[2] Our model rationalizes this observation: a more liquid secondary asset market increases the quality of the collateral (interpreted as the willingness of the lender to accept the asset as collateral), and, thus, it increases the price that borrowers are willing to pay in order to acquire the asset in the first place.

A higher asset supply increases welfare, not only because each unit of the asset bears fruit (we consider the case of a real asset), but also because in any given transaction in the frictional goods market, the buyer can purchase a higher quantity by being able to issue a bigger loan. On the other hand, a higher inflation typically reduces welfare because it increases the holding cost of money and reduces the real balances held by buyers (for any given level of asset supply).

[2] The source of this quote is "A Review of Treasury's Debt Management Policy", June 3, 2002, available at http://www.treas.gov/press/releases/po3149.htm). It should be pointed out that Treasury bills are among the assets that are most widely used as collateral.

Finally, we study the so-called haircut that the lender applies to the collateral asset, i.e., the percentage that is subtracted from the value of the asset that is used as collateral. This term is shown to be increasing both in inflation and in the probability of trading in the OTC market (which can be interpreted as the secondary market liquidity). To see the intuition behind this result, recall that the haircut is given by 1 minus the Loan to Value (LTV) ratio. Hence, anything that tends to increase the value of the collateral asset, will also tend to decrease the LTV ratio, and increase the haircut. Since, as we have already discussed, the inflation and the probability of trade in the OTC market are positively related to the asset price, an increase in either of the two will also lead to an increase in the amount of the haircut. Unlike some other results of the paper (e.g., the positive relationship between asset prices and inflation and the fact that asset prices include liquidity premia), which are supported by findings in the existing empirical literature, the relationship between haircuts and asset market liquidity has not been studied in detail empirically. However, in Section 4.2, we present some evidence which is directly in support of our theoretical findings.

The present paper is part of a large and growing literature that studies the equilibrium prices of assets that can help agents carry out transactions in decentralized markets with imperfect credit. For instance, Geromichalos, Licari, and Suarez-Lledo (2007), Lester, Postlewaite, and Wright (2012), Nosal and Rocheteau (2012), Jacquet and Tan (2012), and Rocheteau and Wright (2012) show (among other results) that financial assets can be priced above their fundamental value, or, equivalently, carry liquidity premia which reflect the assets' ability to facilitate trade in these types of markets. In our opinion, this is an important result because it implies that the traditional view in finance, according to which assets are priced (only) based on the stream of consumption that they yield, might be missing an important element. In fact, a number of recent papers highlight that models in which assets are endowed with liquidity properties can serve as a lens through which one can rationalize many traditional asset pricing puzzles. Most notably, Lagos (2010) shows that the ability of assets to help agents trade in decentralized markets can offer a new perspective for looking at the equity premium and risk-free rate puzzles.

Given this discussion, one can arguably claim that this literature delivers some importnant insights to the study of asset pricing. However, the aforementioned papers are based on the strong assumption that assets serve as media of exchange. This assumption may be subject to criticism, since, in reality, we rarely observe transactions where the buyer pays the seller directly with assets. On the contrary, it is very common to see transactions where the borrower pledges some assets as collateral (the so-called repo market is just one example). The present paper shows that many of the interesting and empirically supported results highlighted in the recent literature do not hinge upon the assumption that assets serve as means of payment. In contrast, we model asset liquidity in a realistic and empirically relevant way, since in our framework assets derive their liquidity properties only through their ability to serve as collateral.[3] Except from the obvious modeling differences, our paper also differs from the aforementioned papers in terms of its asset pricing predictions in ways that we discuss in detail in Section 4.2.

It should be pointed out that modeling asset liquidity in an empirically relevant way is not an end in itself. More importantly, focusing on the role of assets as collateral allows us to study a number of questions that have not been yet addressed in the literature: How does the ability of the asset to serve as collateral affect its price? How does the asset price depend on the frictions of the secondary market where lenders sell off the collateral? Moreover, our paper produces a number of interesting empirical implications, which could be tested in the future. For instance, it would be interesting to examine whether the spread between the yields of T-bills and municipal bonds can, at least in part, be explained by the fact that municipal bonds are known to trade in notoriously illiquid secondary markets (see Green (1993)). Also, our model predicts that prices are affected by the bargaining power of the sellers of assets in the secondary asset market. This is another implication that one might like to take to the data, using "bid-ask spreads" as a proxy for the bargaining power of investors in various OTC asset markets.

In recent work, Ferraris and Watanabe (2011), Li and Li (2013), and Venkateswaran and Wright (2013) also develop models in which assets serve as collateral rather than as media of exchange. However, none of these papers considers the possibility that the lender can visit a secondary asset market in order to sell off the collateral, which is the central idea in our model. Moreover, Carapella and Williamson (2012) study a model of collateral, and they focus on the incentives of borrowers to default (in our analysis this is exogenous). They show that government debt can act to make defaulting on credit contracts more costly, thus relaxing incentive constraints and increasing transactions and welfare.

Finally, the present work is related to a number of papers which build upon the Duffie *et al* framework in order to study asset trade in OTC markets characterized by search and bargaining frictions. Examples of such papers include Weill (2007), Lagos and Rocheteau (2008), Vayanos and Weill (2008), Lagos, Rocheteau, and Weill (2011), Chiu and Koeppl (2011), and Afonso and Lagos (2012). To the best of our knowledge, the present paper is the first to formalize the idea that the degree of liquidity in the OTC market can critically affect welfare by improving the role of certain assets as collateral.

In Section 2, we provide a description of the physical environment. In Section 3, we study the optimal behavior of the agents. In Section 4, we characterize the steady state equilibrium of the model and state the main results of the paper. Section 5 concludes.

[3] Interestingly, Lagos (2011) makes precisely this point. He suggests that as long as an asset can help an untrustworthy buyer obtain what he wants from a seller, either by serving as means of payment or as collateral, that asset will have liquidity properties (implying that it should also be priced accordingly). Despite making this interesting suggestion, the author also studies a model where assets serve as media of exchange. Hence, a contribution of this paper is to formalize Lagos's (2011) suggestion. In recent work, Geromichalos and Herrenbrueck (2012) show that a real asset can carry a liquidity premium even if it does not serve a medium of exchange or a collateral, simply because agents can use it as a hedge against inflation.

2 Physical Environment

Time is discrete with an infinite horizon, and each period consists of three sub-periods characterized by different markets. In the first sub-period, economic activity takes place in a decentralized goods market characterized by bilateral and anonymous trade, as in Lagos and Wright (2005). We refer to it as the LW market. Due to anonymity, buyers cannot pay the sellers with unsecured credit (e.g., an IOU). To purchase some goods a buyer either has to pay the seller with money (in a *quid pro quo* fashion) or make a promise to repay, provided that this promise is backed by some assets that the seller keeps in the form of collateral. During the second sub-period, a secondary asset market opens, which is similar to the Over-the-Counter market of Duffie, Gârleanu, and Pedersen (2005). We refer to it as the OTC market. Agents who accepted assets as collateral in the LW market, and whose borrowers reneged on their debt, can visit the OTC market and (try to) sell the collateral. The third sub-period is a traditional Walrasian or centralized market. We term it the CM. The CM can be thought of as the "settlement market" where agents work, consume, and have access to perfectly competitive markets where they can rebalance their asset holdings in anticipation of the new period which is about to begin.

There are three types of agents, buyers, sellers, and investors. The measure of buyers and sellers is normalized to 1. The measure of investors is not crucial and is discussed later. An agent's identity as a buyer or seller depends on the economic activity that she performs in the LW market. Investors do not participate in the LW market. These agents have an (exogenously given) higher valuation for the collateral asset than sellers, and, hence, an incentive to visit the OTC market and trade with sellers who wish to dispose of the collateral. Buyers are the only long-lived agents in the model. A buyer who is active in period t will remain active in $t+1$ with probability $1 - l \in (0, 1)$. Buyers who die are immediately replaced by a "clone".[4] Although buyers always have the incentive to honor their debts (we focus on incentive compatible contracts), sometimes they will not be able to do so, simply because they are deceased. For reasons that will become clear later, we assume that sellers and investors only live for one period.[5]

Buyers, the only agents who have dynamic considerations, discount future between periods (but not sub-periods) at the rate $\beta \in (0, 1)$. They consume in the first and the third sub-periods,

[4] This is a standard trick used in search theory in order to ensure that the measure of agents (here buyers) remains fixed. For example, see the marriage market model of Burdett and Coles (1997).

[5] Since our goal is to study the pricing properties of assets that serve as collateral, we focus on an environment where the asset demand comes from the agents who actually use the asset as collateral, i.e., the *buyers*. If sellers were long-lived (and did not die with probability l, like buyers do), then they might absorb all the asset supply simply because they are more patient than buyers (the effective discount rate of a buyer is $\beta(1 - l)$, while that of a seller is β). But this would kill off all the interesting asset pricing results that stem from the asset's property to facilitate trade in the LW market. To avoid this, we assume that sellers (and investors) live only for one period. Alternatively, one could assume that sellers and investors are long-lived but they also die with probability l in every period. Then, these agents would choose to not purchase any assets in the CM because they cannot take advantage of the liquidity properties of the asset (for a formal proof, see Rocheteau and Wright (2005) and Geromichalos and Simonovska (2010)). In this case, all the main results of the paper would remain unaltered.

and supply labor in the third sub-period. Their preferences for consumption and labor within a period are given by $\mathcal{U}(X, H, q) = X - H + u(q)$, where X and H represent consumption and labor in the CM, respectively, and q is consumption in the LW market. Sellers consume only in the CM, they may trade assets in the OTC market, and they produce in the LW market and the CM. Their preferences are given by $\mathcal{V}(X, H, h) = X - H - c(h)$, where X and H are as above, and h stands for hours worked in the LW market. Investors also consume only in the CM and trade assets in the OTC market. Their preferences are given by $\mathcal{W}(X, H) = X - H$, where X and H are as above. We assume that u is twice continuously differentiable with $u(0) = 0$, $u' > 0$, $u'(0) = \infty$, $u'(\infty) = 0$, and $u'' < 0$. For simplicity, we set $c(h) = h$, but this is not crucial for any results. Let $q^* \equiv \{q : u'(q^*) = 1\}$ and $q^{**} \equiv \{q : u'(q^{**}) = 1 - l\}$.[6]

We now describe the three sub-periods in more detail. It is helpful to begin the description from the third sub-period. In this period, all agents consume and produce a general good or *fruit*. Agents have access to a technology that transforms one unit of labor into one unit of fruit. There is also a set of trees which produce fruit and live for one-period. The supply of these trees is denoted by $A > 0$, and it is fixed over time. Hence, trees which are born in period t deliver a real dividend (fruit) in the CM of $t + 1$, and then vanish (and get replaced by a new set of trees). Buyers can purchase shares of these trees at the market price ψ_t, which they take as given. Since sellers and investors only live for one period, they never wish to buy assets in the CM (see footnote 5). However, some sellers and investors might enter the CM with assets (and, hence, have a claim to their dividend) acquired during the earlier rounds of trade. Following Duffie *et al*, we assume that the same asset has a different valuation in the hands of different agents. In particular, each unit of asset delivers one unit of fruit to buyers and investors, but it yields $1 - \delta$ units of fruit if held by a seller, with $\delta \in (0, 1]$.[7] This assumption makes the model interesting, since it generates gains from trade in the OTC market, but also realistic, since very often assets that are used as collateral are valued more highly by the borrower than by the lender.

In the CM, buyers can also purchase fiat money, whose market price is denoted by φ_t. Its supply is controlled by a monetary authority, and it evolves according to $M_{t+1} = (1 + \mu)M_t$, with $\mu > \beta(1 - l) - 1$. The benchmark case where $\mu \to \beta(1 - l) - 1$ is the adjusted Friedman rule for our model (adjusted to the fact that buyers in our model effectively discount future at rate $\beta(1 - l)$ rather than just β). New money is introduced (if $\mu > 0$) or withdrawn (if $\mu < 0$) via lump-sum transfers to buyers in the CM. Although money is fiat (i.e., it has no intrinsic value), we assume that it possesses all the properties that constitute it an acceptable means of payment

[6] The meaning of these terms is explained in more detail in Section 3.2. In short, q^* is the amount of good that maximizes the current surplus of a match between a buyer and a seller, i.e., $u(q) - q$, while the q^{**} is the amount that maximizes surplus taking under consideration that the buyer effectively discounts future between the current sub-period (LW market) and the third sub-period (CM) at rate $1 - l$, due to the probability of death.

[7] Duffie *et al* offer several interpretations of the agents with low valuations. More precisely, they claim that "[...] a low-type investor may have (i) low liquidity (that is, a need for cash), (ii) high financing costs, (iii) hedging reasons to sell, (iv) a relative tax disadvantage, or (v) a lower personal use of the asset". In our story, the leading interpretation is (v), although some of the other interpretation (like (i),(ii), or (iii)) might also be relevant.

in the LW market (i.e., it is portable, storable, and recognizable by all agents).

The first sub-period is a standard decentralized goods market *à la* Lagos and Wright (2005). Buyers and sellers meet in a bilateral fashion and negotiate over the terms of trade. Buyers can purchase goods using money, but if a buyer's money holdings are not enough to purchase the desired amount of good, she can use the asset as collateral in order to obtain a secured loan from the seller and increase her consumption. In return, the buyer promises to deliver to the seller a certain amount of the general good in the forthcoming CM.[8] We focus on incentive compatible contracts, which guarantee that a surviving buyer will choose to repay her debt. If the buyer dies, all her assets will be seized by the seller.[9] The vanishing of buyers takes place after they leave the LW market, and deceased buyers are replaced by their clones in the current period's CM (buyers do not participate in the OTC market). Sellers who have given loans to deceased buyers are immediately notified, and they can enter the OTC in order to sell the collateral. Since most of the interesting results of the paper follow from the interaction of agents in the OTC market, we keep the LW market setup as simple as possible by assuming that all buyers match with a seller (and vice versa) and make a take-it-or-leave-it (TIOLI) offer to her.

Finally, consider the second sub-period. Sellers who granted loans to buyers who later deceased, and whose measure equals l, seize the assets. Given the different asset valuation between sellers and investors (the term $\delta > 0$), there are certain gains from trade to be exploited in the OTC market.[10] Letting ι denote the measure of investors, we assume that a matching function $f(l, \iota) \leq \min\{l, \iota\}$ brings together sellers and investors in the OTC, where f is homogeneous of degree one and increasing in both arguments. Within each match, the terms of trade are determined through proportional bargaining, following Kalai (1977), where $\lambda \in (0, 1)$ represents the sellers' bargaining power. In contrast to the LW market, we assume that the OTC market is not characterized by anonymity and imperfect credit.[11] Hence, neither a medium of exchange nor collateral is needed to facilitate transactions in this market. When an investor purchases assets from a seller, she can pay with unsecured credit, i.e., an unbacked promise to deliver a certain amount of fruit in the forthcoming CM. This, in turn, implies that all profitable

[8] It should be pointed out that agents are free to use the asset as a medium of exchange (as they do with money), but they choose not to because the asset has a bad property as a means of payment: it is worth more to the buyer than it is to the seller. Hence, agents rationally choose to use the asset as collateral in the way described above.

[9] The fact that all the assets go to the possession of the seller in case of the buyer's death is a *result* rather than an assumption. The only assumption made here is that buyers do not obtain any utility from leaving bequests to the clones who replace them. Given this, it is easy to see that buyers are happy to leave their assets to the seller in the case of death, since, this allows them to purchase more good in the LW market now (see equation (8)), and it has no cost later (they do not care what happens to the assets when they are dead).

[10] The analysis would not change significantly if we assumed that sellers trade assets in the OTC with (surviving) buyers rather than investors. The reason for introducing a third type of agents, namely the investors, is simply to highlight that the agents with whom sellers trade in the LW and the OTC markets are typically not the same.

[11] This assumption seems to be realistic for most real-world asset markets. In terms of modelling, frictions such as anonymity in the LW market are necessary in order to generate a role for a medium of exchange or collateral in that market. Assuming that trade in the OTC market is also anonymous would only add unnecessary complications without delivering many interesting economic insights.

trades will always be consummated in the OTC, as opposed to the LW market, where buyers may not be able to purchase the desired quantity of good due to liquidity and credit constraints.

3 Value Functions and Optimal Behavior

3.1 Value Functions

We begin with the description of the value functions of the typical buyer who is the agent that makes all the interesting decisions in the model. Consider first the value function of a buyer who enters the CM with money and asset holdings (m, a), and with a debt (to a seller with whom she matched in the LW market) equal to b. Notice that if the buyer in question is a newly born clone, then $m = a = b = 0$. The Bellman's equation is

$$W(m, a, b) = \max_{X,H,\hat{m},\hat{a}} \{X - H + \beta V(\hat{m}, \hat{a})\}$$
$$\text{s.t.} \quad X + \varphi\hat{m} + \psi\hat{a} = H + \varphi(m + \mu M) + a - b,$$

where variables with hats denote next period's choices, and V represents the buyer's value function in the LW market. Replacing $X - H$ from the budget constraint into W yields

$$W(m, a, b) = \varphi m + a - b + \varphi\mu M + \max_{\hat{m},\hat{a}} \{-\varphi\hat{m} - \psi\hat{a} + \beta V(\hat{m}, \hat{a})\}. \tag{1}$$

As is standard in models that build on Lagos and Wright (2005), the optimal choice of the agent does not depend on her current asset holdings (due to the linearity of $\mathcal{U}$). As a result, the CM value function is linear. Hence, define $\Lambda \equiv \varphi\mu M + \max_{\hat{m},\hat{a}} \{-\varphi\hat{m} - \psi\hat{a} + \beta V(\hat{m}, \hat{a})\}$, and write

$$W(m, a, b) = \Lambda + \varphi m + a - b. \tag{2}$$

Now consider the problem of a seller in the CM. Since this agent will vanish once the CM sub-period is over, she will never purchase any assets in that market, but she will typically enter the CM with some money, m, which she received as a means of payment in the preceding LW market. Moreover, if the seller was matched in the LW market with a buyer who did not renege on her debt, she will be credited with b units of fruit. It is straightforward to check that, for any (m, b), this seller's CM value function is[12]

$$W^{SN}(m, b) = \varphi m + b. \tag{3}$$

[12] More formally, this seller's problem is to choose X, H in order to maximize her net utility $X - H$, subject to the budget constraint $X = H + \varphi m + b$. Replacing the term $X - H$ into W^{SN} yields the expression reported in (3).

Consider now a seller who was matched in the LW market with a buyer who later vanished. Except from the money, m, which she collected as payment during the LW trade, this seller might also hold some assets, a, which she acquired due to the buyer's default. Also, she might have sold some assets in the OTC market, in return for some credit, c, to be delivered in the current CM.[13] It is easy to check that, for any (m, a, c), this seller's CM value function is

$$W^{SD}(m, a, c) = \varphi m + a(1 - \delta) + c. \tag{4}$$

Lastly, consider the typical investor in the CM. Like sellers, investors do not purchase any assets in this market. When an investor enters the CM, she can only hold assets, a, which she bought from a seller in the OTC market, and in return for which she promised to deliver c units of fruit. It can be easily verified that, for any (a, c), the investor's CM value function is

$$W^{I}(a, c) = a - c. \tag{5}$$

Next, consider the value functions in the LW market. Let q denote the quantity of good produced by the seller, d the amount of money that the buyer pays on the spot, and b the amount of fruit that she promises to deliver to the seller in the forthcoming CM. These terms are determined through bargaining in Section 3.2. The LW value function for a buyer who enters that market with a portfolio (m, a) is

$$V(m, a) = u(q) + (1 - l)W(m - d, a, b), \tag{6}$$

where $1 - l$ is the probability of survival. The LW value function for a seller (who holds no money or assets) is[14]

$$V^S = -q + l\Omega^S(d, a) + (1 - l)W^{SN}(d, b),$$

where $\Omega^S(d, a)$ denotes the value function of a seller who enters the OTC market with money holdings d and asset holdings a.

To finish this sub-section, consider the value functions in the OTC market. The agents who participate here are investors (with measure ι) and sellers who acquired some collateral (with measure l). Given the matching function f, the matching probability for sellers and investors

[13] As a clarification, notice that the term c is a repayment promise from an investor (in the OTC market) while b is a repayment promise from a buyer (in the LW market). Recall that the former is an unbacked promise while the latter is a secured (collateralized) promise, due to the assumption that, unlike the OTC, the LW market is characterized by anonymous trade.

[14] With probability l the seller is matched with a buyer who will vanish after the LW market closes. This seller will seize the buyer's asset holdings, and she will attempt to sell them in the OTC market. With probability $1 - l$ the seller is matched with a buyer who honors her debt. This seller proceeds to the CM with money holdings d and a credit of b units of fruit.

is given by $\alpha_S = f(l,\iota)/l$ and $\alpha_I = f(l,\iota)/\iota$, respectively. Let χ denote the units of asset that the seller transfers to the investor, and c the amount of fruit that the investor promises to deliver to the seller in the forthcoming CM (these terms will also be determined through bargaining in Section 3.2). The OTC value function for a seller who enters that market with portfolio (m,a) is

$$\Omega^S(m,a) = \alpha_S W^{SD}(m, a-\chi, c) + (1-\alpha_S) W^{SD}(m,a,0). \tag{7}$$

The OTC value function for an investor (who enters with no money or assets) is

$$\Omega^I = \alpha_I W^I(\chi, c) + (1-\alpha_I) W^I(0,0).$$

We now describe the determination of the terms of trade in the various markets.

3.2 Bargaining Problems

We proceed by backwards induction and study the bargaining problem in the OTC market first. Consider a meeting between a seller with portfolio (m,a), and an investor who holds no assets at this stage. As in Section 3.1, χ represents the amount of assets that are transferred to the investor, and c denotes the amount of goods that the investor promises to deliver to the seller in the CM. Following Kalai's "proportional" bargaining solution, and letting $\lambda \in (0,1)$ denote the seller's bargaining power, we can write the bargaining problem as

$$\max_{\chi,c} \left\{ W^{SD}(m, a-\chi, c) - W^{SD}(m,a,0) \right\}$$
$$\text{s.t.} \quad W^{SD}(m, a-\chi, c) - W^{SD}(m,a,0) = \tfrac{\lambda}{1-\lambda} \left[W^I(\chi, c) - W^I(0,0) \right],$$

and the feasibility constraint $\chi \leq a$. That is, the proportional bargaining solution maximizes the seller's surplus subject to the constraint that this surplus equals a fixed proportion (i.e., the term $(1-\lambda)/\lambda$) of the investor's surplus, and the feasibility constraint (for a more detailed analysis of proportional bargaining in monetary theory, see Borağan Aruoba, Rocheteau, and Waller (2007)).

Exploiting the linearity of the CM value fucntions (i.e., (4) and (5)), and substituting the term $c - \chi(1-\delta)$ from the constraint into the objective function, simplifies the problem to

$$\max_{\chi,c} \{\lambda \chi \delta\}$$
$$\text{s.t. } c = \chi(1-\delta) + \lambda \chi \delta,$$

and $\chi \leq a$. The last expression is fairly intuitive: solving the proportional bargaining problem is equivalent to maximizing the total surplus of the match, $\chi\delta$, subject to the constraint that a fraction λ of the (maximized) surplus goes to the seller, and the remaining surplus goes to the investor. Clearly, the total surplus of the match is equal to the units of asset that change hands, χ, times the surplus generated every time a unit of asset goes from the hands of low-valuation type (the seller) into the hands of the high-valuation type (the investor), δ. The following lemma describes the solution to this problem.

Lemma 1. *The solution to the bargaining problem is given by $\chi = a$, and $c = (1 - \delta + \delta\lambda)a$.*

Proof. The proof is straightforward, and it is, therefore, omitted. □

Since the OTC market is characterized by perfect credit, the buyer of assets (investor) will never be constrained, and all profitable trades will be consummated. Put simply, the seller will hand over all her assets to the investor since for each unit of assets that changes hands a constant surplus, $\delta > 0$, is generated. Then, the bargaining protocol simply determines c in order to satisfy the sharing rule of the surplus. It is easy to check that, under the suggested bargaining solution, the surplus of the seller, given by $c - \chi(1 - \delta)$, is equal to $\lambda\delta a$, i.e., a fraction λ of the total surplus generated by asset trade.

We now study the terms of trade in a LW market meeting. Consider a match between a buyer with a portfolio (m, a), and a seller who carries no assets at this stage. The two parties negotiate over the quantity q produced by the seller, the units of money d that the buyer pays on the spot, and the amount of fruit b that she promises to deliver to the seller in the CM. Due to anonymity such a promise has to be backed by collateral. If the buyer fails to honor her debt, the contract between the two parties specifies that all the assets a will go to the possession of the seller.[15] The buyer makes a TIOLI offer, maximizing her surplus subject to the seller's participation constraint, the cash constraint, and the incentive compatibility constraint. The bargaining problem can be described by

$$\max_{d,b,q} \{u(q) + (1 - l)[W(m - d, a, b) - W(m, a, 0)]\}$$
$$\text{s.t.} \quad -q + l\Omega^S(d, a) + (1 - l)W^{SN}(d, b) - W^{SN}(0, 0) = 0,$$

$d \leq m$, and $0 \leq b \leq a$. Notice that a seller who matches with a buyer who defaults (with probability l) goes to the OTC market with a units of the asset, while a seller who matches with a buyer that honors her debt (with probability $1 - l$) continues directly to the CM with the d dollars that she already received plus a credit equal to b units of fruit. Given the anonymity in the LW market, the seller cannot track down the buyer and force her to deliver the b units of fruit. However, the constraint $b \leq a$ guarantees that a (surviving) buyer will always have the incentive to repay, otherwise she will lose her assets which entitle her to a units of fruit, an

[15] The buyer is happy to sign this contract. If she dies, she does not care who gets the assets, while promising these assets to the seller induces the latter to work harder now and produce a higher q. See also footnote 9.

amount that exceeds her debt. The term $W^{SN}(0,0)$ is simply the threat point of the seller.

One can now substitute the value functions W, W^{SN}, and Ω^S from equations (2), (3), and (7) into the bargaining problem. The function Ω^S will contain the term W^{SD}, which we can substitute from (4). Moreover, having already solved for the OTC terms of trade, we know that any time a seller matches in the OTC, she will give away all her assets (i.e., $\chi = a$), and she will receive a credit from an investor equal to $c = (1-\delta+\delta\lambda)a$. Exploiting all these pieces of information, we can re-write the bargaining problem as

$$\max_{d,b,q} \{u(q) - (1-l)(\varphi d + b)\}$$
$$\text{s.t.} \quad -q + l\{\varphi d + \big[1-\delta(1-\alpha_S\lambda)\big]a\} + (1-l)(\varphi d + b) = 0, \tag{8}$$

and the cash and loan constraints $d \leq m$ and $0 \leq b \leq a$.

Notice that the seller's participation constraint (8) can be re-written as

$$\begin{aligned} q &= \varphi d + (1-l)b + l[\alpha_S(1-\delta+\lambda\delta) + (1-\alpha_S)(1-\delta)]a = \\ &= \varphi d + (1-l)b + l(1-\delta+\alpha_S\lambda\delta)a. \end{aligned}$$

This expression is intuitive. For her cost of producing the good (q), a seller should be compensated with money (whose real value equals φd), with a promise of b units of fruit if the buyer survives, and with the value of the asset as collateral, if the buyer vanishes. The latter is given by $[\alpha_S(1-\delta+\lambda\delta)+(1-\alpha_S)(1-\delta)]a$, which simplifies to $l(1-\delta+\alpha_S\lambda\delta)a$: if the seller does not match in the OTC (with probability $1-\alpha_S$), she will have to hold on to the collateral and receive the lower dividend of $1-\delta$ per share. If she matches in the OTC, we know from Lemma 1, that she will give up all her assets, but in return she will be credited with $(1-\delta+\delta\lambda)a$ units of fruit in the forthcoming CM. It turns out that the term $l(1-\delta+\alpha_S\lambda\delta)$ will appear very frequently and play an important role in the analysis. Hence, in order to save on notation, we define

$$x \equiv l(1-\delta+\alpha_S\lambda\delta).$$

The following lemma describes the solution to the bargaining problem in the LW market.

Lemma 2. *Let $\pi(m,a) \equiv \varphi m + xa$, and define the following regions of joint money and asset holdings of the buyer:*

$$\begin{aligned} &\text{In Region 1, } (m,a) \text{ satisfies } \pi(m,a) \geq q^{**}, \\ &\text{In Region 2, } (m,a) \text{ satisfies } q^* \leq \pi(m,a) < q^{**}, \\ &\text{In Region 3, } (m,a) \text{ satisfies } q^* - (1-l)a \leq \pi(m,a) < q^*, \\ &\text{In Region 4, } (m,a) \text{ satisfies } \pi(m,a) < q^* - (1-l)a. \end{aligned}$$

Then, for any price φ the solution to the bargaining problem is

$$d(m,a) = \begin{cases} \frac{1}{\varphi}\big(q^{**} - xa\big), & \text{if } (m,a) \in \text{Region } 1 \\ m, & \text{if } (m,a) \in \text{Region } 2, 3 \text{ or } 4 \end{cases}$$

$$q(m,a) = \begin{cases} q^{**}, & \text{if } (m,a) \in \text{Region } 1 \\ \pi(m,a), & \text{if } (m,a) \in \text{Region } 2 \\ q^{*}, & \text{if } (m,a) \in \text{Region } 3 \\ \pi(m,a) + (1-l)a, & \text{if } (m,a) \in \text{Region } 4 \end{cases}$$

$$b(m,a) = \begin{cases} 0, & \text{if } (m,a) \in \text{Region } 1 \text{ or } 2 \\ \frac{q^{*} - \pi(m,a)}{1-l}, & \text{if } (m,a) \in \text{Region } 3 \\ a, & \text{if } (m,a) \in \text{Region } 4 \end{cases}$$

Recall that the terms q^* and q^{**} satisfy $u'(q^*) = 1$ and $u'(q^{**}) = 1 - l$, respectively.

Proof. See the appendix. □

The various regions listed in the lemma are depicted in Figure 1. These regions are determined by the values of the terms $\pi(m,a) \equiv \varphi m + xa$ and $\pi(m,a) + (1-l)a$. The first captures the buyer's real purchasing power excluding credit, i.e., it includes the (real) units of money that change hands on the spot, φm, and the expected value of the assets that the seller will acquire (if the buyer vanishes), which, exploiting Lemma 1, simplifies to xa. The second term captures the total purchasing power of the buyer, i.e., the one that includes the maximum loan that she can obtain after pledging all of her assets a as collateral.[16] Notice that the purchasing power in any Region $i \in \{1, 2, 3, 4\}$ is generally decreasing in i. Finally, notice that the quantity q^* is the one that maximizes the spontaneous surplus of a match, $u(q) - q$. However, in our environment the buyer might vanish before the CM sub-period starts, thus, effectively she behaves as if she discounted the future at rate $1 - l$. This, in turn, means that if the buyer can afford it (in a sense to be made precise below), she will purchase the quantity $q^{**} > q^*$.

Lemma 2 is quite intuitive. In Region 4, the buyer's (m, a) holdings are so low that, even after obtaining the maximum possible loan, she cannot afford to purchase q^*. In this case, the buyer spends all her money ($d = m$) and pledges all her assets ($b = a$), which allows her to obtain $q = \pi(m,a) + (1-l)a$. As we enter Region 3, the buyer's (m, a) holdings become more

[16] In papers such as Geromichalos *et al* (2007) and Jacquet and Tan (2010), money and assets are (perfect) substitutes, and the amount of good that the buyer can purchase in the LW market is simply an increasing function of the value of money and assets summed together. Our bargaining problem has a much richer solution. It depends on the value of money that the seller receives on the spot, and the value of the asset that she rationally expects to receive by selling it in the OTC market, which, in turn, depends on the *micro-structure* of that market.

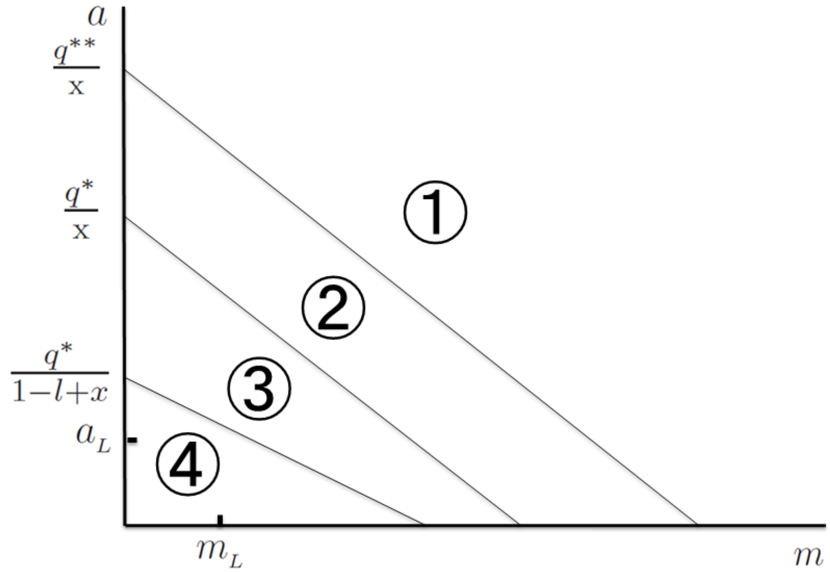

Figure 1: Regions of the bargaining solution.

plentiful, and the buyer can afford to purchase more good. Interestingly, the bargaining solution reveals that this will not be the case: as the buyer is already able to purchase q^*, the benefit of increasing q a little more, while still pledging all of her assets, becomes smaller than the benefit that she can obtain by reducing her debt. As a result, in Region 3, instead of increasing q the buyer prefers to deleverage by reducing b.[17] This result differs significantly from the existing literature, since in papers where money and assets compete as media of exchange, q is increasing in the buyer's purchasing power (see also footnote 16). As (m, a) increase further, we enter Region 2, and the buyer can afford to purchase q^* without obtaining a loan (i.e., $\pi(m, a) \geq q^*$). In this case, $b = 0$, and since the benefit of reducing her debt has been fully exploited, the buyer uses the extra resources to bring q closer to the first-best q^{**}. Finally, as we enter Region 1, the buyer can afford to purchase q^{**} without obtaining any loan. Here, the buyer's (m, a) holdings are so plentiful that any further increase has no effect on the terms of trade.[18]

[17] To see this point more formally, assume that initially $(m, a) = (m_0, a_0)$, satisfying $\pi(m_0, a_0) + (1 - l)a_0 = q^*$, so that $q(m_0, a_0) = q^*$, $b(m_0, a_0) = a_0$, and $d(m_0, a_0) = m_0$. Now consider an increase of assets to $a' = a_0 + \epsilon$, while money holdings remain unaltered, and focus on the following two plans of action for the buyer. Under Plan A the buyer still pledges all of her assets in an attempt to keep her consumption of q at the highest possible level. Under this scenario $b_A(m_0, a') = a'$ and $q_A(m_0, a') = \pi(m_0, a') + (1 - l)a' = q^* + \epsilon(1 - l + x)$. Under Plan B the buyer keeps her consumption at q^* and uses the extra units of asset in order to decrease the value of her loan b. It is easy to check that in this case the value of the loan satisfies $(1 - l)b_B(m_0, a') = q^* - \varphi m_0 - xa'$. Defining $\tilde{\epsilon} \equiv \epsilon(1 - l + x) > 0$, it is now straightforward to verify that the increase in surplus that the buyer can achieve by following Plan B is greater than the one that she can achieve under Plan A, if and only if $u(q^*) + \tilde{\epsilon} > u(q^* + \tilde{\epsilon})$. Of course, this inequality is always satisfied since $u'(q) < 1$ for all $q > q^*$.

[18] Unlike the assets of deceased buyers, which will go to the possession of the seller with whom that buyer

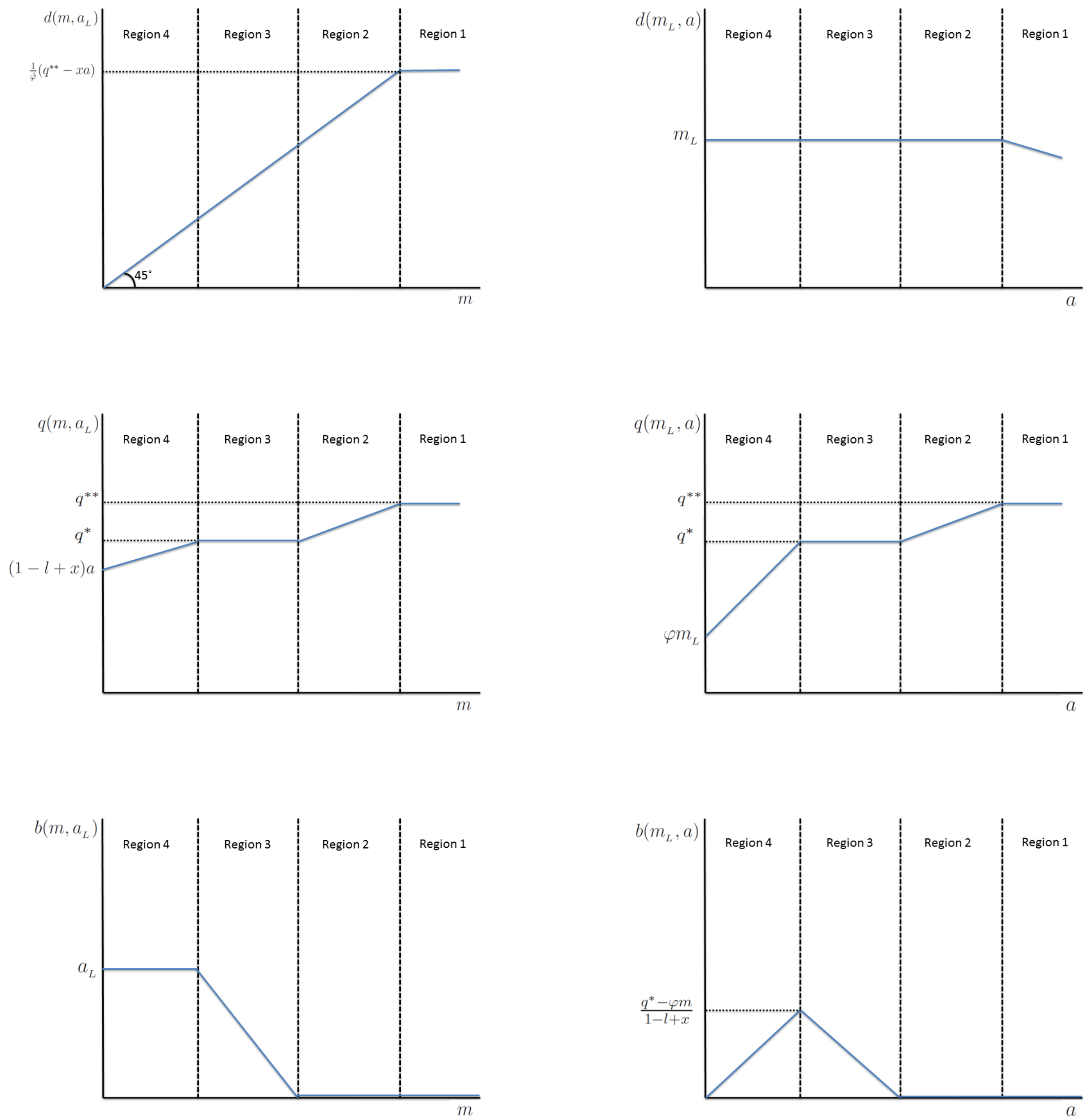

Figure 2: The bargaining solution.

Figure 2 summarizes these results. The left panel depicts the bargaining solutions d, q, b as functions of m, for fixed $a = a_L$, and the right panel depicts the bargaining solutions as functions of a, for fixed $m = m_L$. Notice that the values a_L, m_L (depicted on Figure 1) have been set small enough so that all four regions are relevant. Consider the left panel of Figure 2. Given

matched, we have not assumed that the same will happen with the money of deceased buyers. Hence, a question that arises is what happens to the money of deceased buyers. However, this issue will never arise in equilibrium: when it is costly to carry money (as we assume is the case here), buyers will never carry money holdings that bring them in the interior of Region 1.

$a = a_L$, the buyer spends all her money ($d = m$) up to the point that she enters Region 1. Within that region carrying more money has no effect on the terms of trade. The term $q(m, a_L)$ is strictly increasing in m within Regions 2 and 4. It is constant within Region 1 for the same reason as above (i.e., carrying more money has no effect on the solution), and also within Region 3, because the buyer prefers to deleverage rather than increase her consumption. Finally, $b(m, a_L) = a_L$ in Region 4, since with such low purchasing power the buyer chooses to pledge all of her assets. In Region 3 the buyer takes advantage of her higher purchasing power and reduces b, up to the point where she enters Region 2, and she is able to purchase $q(m, a_L) \geq q^*$ without need to get any credit (so that $b(m, a_L) = 0$). The functions $d(m_L, a), q(m_L, a), b(m_L, a)$ depicted on the right panel of Figure 2 admit similar interpretations.

3.3 Objective Functions and Optimal Behavior

A benefit of adopting the Lagos-Wright framework is that, regardless of their trading history, all buyers will choose the same money and asset holdings $(\hat{m}, \hat{a})$ (this is true even for a buyer who was just born in order to "replace" a deceased buyer). This choice is described by the buyer's optimal behavior, which, in turn, can be derived by maximizing the buyer's "objective function". To obtain the objective function, substitute (6) into (1), and focus on the term inside the maximum operator (i.e., ignore the terms that do not affect the choice variables). Defining the objective function as $J(\hat{m}, \hat{a})$, after some manipulations one we can show that

$$J(\hat{m}, \hat{a}) = -\varphi\hat{m} - \psi\hat{a} + \beta\big\{u(q(\hat{m}, \hat{a})) + (1-l)\big[\hat{\varphi}(\hat{m} - d(\hat{m}, \hat{a})) + \hat{a} - b(\hat{m}, \hat{a})\big]\big\},$$

where the first two terms represent the cost of purchasing $\hat{m}$ units of money and $\hat{a}$ units of the asset, and the last term captures the expected, discounted benefit of a buyer who holds a portfolio $(\hat{m}, \hat{a})$. Clearly, the latter depends on the terms q, d, b, which are determined by the solution to the bargaining problem in the LW market. Hence, given her own choices of $(\hat{m}, \hat{a})$, the buyer can find herself in different regions of the bargaining solution. Letting $J^i(\hat{m}, \hat{a})$ denote the objective function in Region i, $i \in \{1, 2, 3, 4\}$, and exploiting Lemma 2, one can show that

$$\begin{aligned}
J^1(\hat{m}, \hat{a}) &= -\varphi\hat{m} - \psi\hat{a} + \beta\big\{u(q^{**}) + (1-l)\big[\hat{\varphi}(\hat{m} - m^{**}) + \hat{a}\big]\big\},\\
J^2(\hat{m}, \hat{a}) &= -\varphi\hat{m} - \psi\hat{a} + \beta\big[u\big(\hat{\varphi}\hat{m} + x\hat{a}\big) + (1-l)\hat{a}\big],\\
J^3(\hat{m}, \hat{a}) &= -\varphi\hat{m} - \psi\hat{a} + \beta\big[u(q^*) - q^* + \hat{\varphi}\hat{m} + (1-l+x)\hat{a}\big],\\
J^4(\hat{m}, \hat{a}) &= -\varphi\hat{m} - \psi\hat{a} + \beta u\big(\hat{\varphi}\hat{m} + (1-l+x)\hat{a}\big),
\end{aligned}$$

where we have defined $m^{**} \equiv \hat{\varphi}^{-1}\big(q^{**} - x\hat{a}\big)$, i.e., m^{**} denotes the amount of money which, for any given choice of $\hat{a}$, allows the buyer to purchase q^{**} without obtaining a loan.

The next lemma describes the optimal behavior of the representative buyer.

Lemma 3. *Letting $J^i_k(\hat{m},\hat{a}), k=1,2$, represent the derivative of the objective function in Region $i = 1,2,3,4$ with respect to the k-th argument, we obtain*

$$J^1_1(\hat{m},\hat{a}) = -\varphi + \beta(1-l)\hat{\varphi} \tag{9}$$
$$J^1_2(\hat{m},\hat{a}) = -\psi + \beta(1-l) \tag{10}$$
$$J^2_1(\hat{m},\hat{a}) = -\varphi + \beta\hat{\varphi}u'\big(\hat{\varphi}\hat{m} + x\hat{a}\big) \tag{11}$$
$$J^2_2(\hat{m},\hat{a}) = -\psi + \beta\big[u'\big(\hat{\varphi}\hat{m} + x\hat{a}\big)x + 1 - l\big] \tag{12}$$
$$J^3_1(\hat{m},\hat{a}) = -\varphi + \beta\hat{\varphi} \tag{13}$$
$$J^3_2(\hat{m},\hat{a}) = -\psi + \beta(1-l+x) \tag{14}$$
$$J^4_1(\hat{m},\hat{a}) = -\varphi + \beta\hat{\varphi}u'\big(\hat{\varphi}\hat{m} + (1-l+x)\hat{a}\big) \tag{15}$$
$$J^4_2(\hat{m},\hat{a}) = -\psi + \beta u'\big(\hat{\varphi}\hat{m} + (1-l+x)\hat{a}\big)(1-l+x) \tag{16}$$

Moreover, taking the prices $(\varphi,\hat{\varphi},\psi)$ as given, the optimal choice of the representative buyer satisfies:

a) If the optimal choice is strictly within any region, it satisfies $\nabla J(\hat{m},\hat{a}) = \mathbf{0}$, where the various partial derivatives are given by equations (9)-(16).

b) If $\varphi > \beta\hat{\varphi}(1-l)$ and $\psi = \beta(1-l)$, any bundle $(\hat{m},\hat{a})$ with $\hat{m}=0$ and $\hat{a} \geq q^{**}/x$ is optimal.

c) If $\varphi > \beta\hat{\varphi}(1-l)$ and $\psi > \beta(1-l)$, the optimal choice is unique in Regions 2 and 4 but it is indeterminate in Region 3.

Proof. See the appendix. □

We provide an intuitive interpretation of the optimal behavior and relegate the formal proof to the appendix. We will refer to the price $\psi = \beta(1-l)$ as the "fundamental value" of the asset. This term is justified by the fact that this is the unique equilibrium price that the asset would obtain in a world where the asset serves only as a store of value. Clearly, when $\psi = \beta(1-l)$ the cost of carrying the asset across periods is zero and, hence, it would be suboptimal for the buyer to find herself in a region where she cannot afford to purchase q^{**}. If, moreover, the cost of holding money is positive (i.e., $\varphi > \beta\hat{\varphi}(1-l)$), the agent would choose to not hold any money $\hat{m}=0$, and carry any amount of assets that exceeds the level that purchases q^{**}, i.e., $\hat{a} \geq q^{**}/x$. If $\psi > \beta(1-l)$, then carrying the real asset is also costly. The optimal choice of the buyer lies within Regions 2, 3, or 4, and it is characterized by the first-order conditions.

Part (c) of Lemma 3 indicates that within Region 3 the optimal asset and money holdings will be indeterminate. To illustrate this result, in Figure 3 we plot the buyer's money demand for a given of $\hat{a} = a_L$. The plot in the bottom of the figure makes it easy to see which region of the bargaining protocol the buyer will find herself in for various choices of $\hat{m}$. The vertical axis

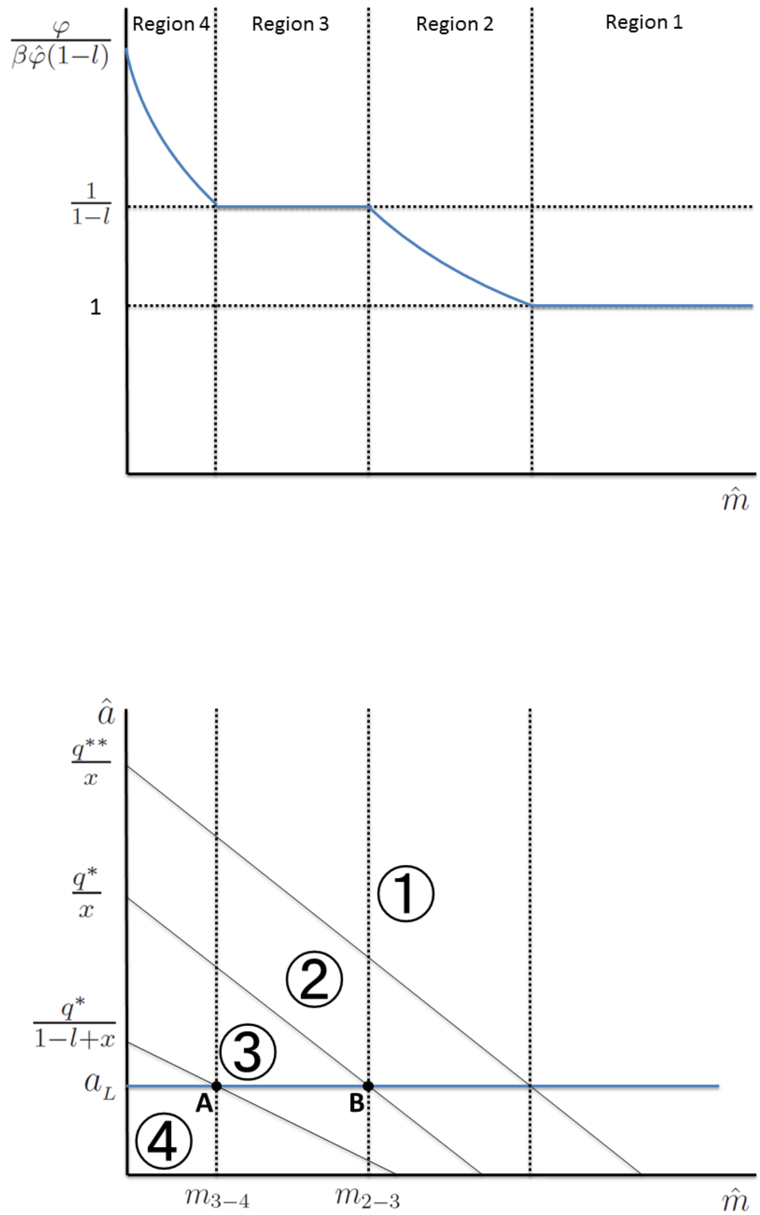

Figure 3: Money demand for $\hat{a} = a_L$.

of the top panel measures the holding cost of money, $\varphi/(\beta(1-l)\hat{\varphi})$.[19] The indeterminacy result stated in Lemma 3 is highlighted by the fact that the demand for money is flat within Region 3 (when $\varphi = \beta\hat{\varphi}$). This is true because in Region 3 the cost of obtaining a loan (increasing b) is the

[19] More precisely, the net cost of holding one unit of money for the buyer is given by the term $-\varphi+\beta\hat{\varphi}(1-l)$. This cost will be positive if and only if $\varphi/(\beta(1-l)\hat{\varphi}) > 1$, which, as we clarified in Section 2, is a maintained assumption of the model. The limiting case where $\varphi/(\beta(1-l)\hat{\varphi}) \to 1$ is simply the modified Friedman rule (modified due to the fact that buyers in our model die with probability l within periods).

same as the cost of holding money. As a result, the buyer is indifferent between holding $(\hat{m}, \hat{a}) = (m_{2-3}, a_L)$ (point B in the figure) without any debt, or holding $(\hat{m}, \hat{a}) = (m_{3-4}, a_L)$ (point A in the figure) with the maximum debt that her collateral allows, i.e., $b = a_L$.[20] Within Regions 2 and 4 the money demand has the standard negative slope. This slope is greater in Region 4 because within that region the buyer is more severely constrained, so that an additional unit of money has a larger marginal benefit (given the concavity of u).

4 Equilibrium

4.1 Definition, Existence, and Uniqueness

In this section we describe the steady state equilibrium of the model. Since, in steady state, the real money balances do not change over time, we have $\varphi M = \hat{\varphi}\hat{M}$, which implies $\varphi/\hat{\varphi} = 1 + \mu$.

Definition 1. A steady state equilibrium is a list $\{\psi, z, q, b, \chi, c\}$, where $z = \varphi M$ represents the real money balances. The equilibrium objects are such that:

i. The representative buyer behaves optimally under the equilibrium prices ψ, φ, and, moreover, $\varphi/\hat{\varphi} = 1 + \mu$.

ii. The equilibrium quantity produced in the LW market, q, is

$$q(z) = \begin{cases} q^{**}, & \text{in Region 1} \\ \tilde{q}_2(z), & \text{in Region 2} \\ q^{*}, & \text{in Region 3} \\ \tilde{q}_4(z), & \text{in Region 4} \end{cases}$$

where $\tilde{q}_2(z) \equiv z + xA$ and $\tilde{q}_4(z) \equiv z + (1 - l + x)A$. The equilibrium value of the loan, b, is

$$b(z) = \begin{cases} 0, & \text{in Regions 1, 2} \\ \tilde{b}(z), & \text{in Region 3} \\ A, & \text{in Region 4} \end{cases}$$

where $\tilde{b}(z) \equiv (1 - l)^{-1}(q^* - z - xA)$.

[20] The terms m_{2-3}, m_{3-4} are indicated in Figure 3 as the levels of money holdings that bring the buyer right on the boundary of Regions 2,3 (m_{2-3}) and 3,4 (m_{3-4}), given $\hat{a} = a_L$. Then, from Lemma 2 we know that when $(\hat{m}, \hat{a}) = (m_{2-3}, a_L)$ the buyer has managed to bring her loan down to zero (this is the very definition of entering Region 2). On the other hand, with asset holdings $(\hat{m}, \hat{a}) = (m_{3-4}, a_L)$, the buyer is so constrained that she is still pledging all of her assets as collateral. Clearly, the buyer is not only indifferent between points A and B, but also between any other portfolio $(\hat{m}, a_L)$, $\hat{m} \in [m_{3-4}, m_{2-3}]$, which will be combined with a loan equal to $b = (1 - l)^{-1}[q^* - \hat{\varphi}\hat{m} - l(1 - \delta + \alpha_s \lambda\delta)a_L]$ (so that the buyer purchases q^* in all cases).

iii. The terms of OTC trade (χ, c) satisfy $\chi = A$, and $c = (1 - \delta + \delta\lambda)A$.

iv. Markets clear and expectations are rational: $\hat{m} = (1 + \mu)M$, and $\hat{a} = A$.

Lemma 4. *A steady state equilibrium $\{\psi, z, q, b, \chi, c\}$ exists and $\{\psi, q, \chi, c\}$ are unique. There exists a region of the parameter space where z and b are indeterminate within a certain range, otherwise z and b are also unique.*

Proof. See the appendix. □

The definition of equilibrium is straightforward. The equilibrium quantity, q, and the size of the loan that the buyer obtains in the LW market, b, are determined by the combination of z and A, and specifically, by the region in which the representative buyer will find herself, given the exogenous value of A and the endogenous value of z. Following Lemma 1, and the fact that, in equilibrium, $a = A$, the quantity of the traded asset, χ, and the amount of credit that the seller gets in return, c, are fully determined by the exogenous asset supply A. Lemma 4 establishes the existence of equilibrium. The equilibrium is generally unique, with the exception of the terms z, b, which will be indeterminate if equilibrium lies in Region 3. This result follows quite naturally, given the indeterminacy of money demand that we discussed earlier (see Figure 3).

4.2 Characterization of Equilibrium

In this section, we wish to describe the equilibrium values of the model's key variables as functions (only) of parameters, specifically, the exogenous asset supply, A, and the policy parameter, μ, which, in equilibrium pins down the holding cost of money. Thus, we first need to replace the endogenous z with the exogenous μ. This task becomes easier with the help of Figure 4.

First, if $A \geq q^{**}/x$, equilibrium lies in Region 1. Here the asset supply is so plentiful that the buyer can purchase q^{**} just by promising to deliver the assets to the seller in the case of death (i.e., the buyer does not even obtain a loan). Moreover, notice that equilibrium will lie in Region 4 if and only if $\mu > \beta - 1$, and it will lie in Region 2 if and only if $\mu < \beta - 1$. Interestingly, this also implies that Region 3 is associated with one and only one value of μ: $\mu = \beta - 1$. Hence, while in earlier figures (e.g., Figures 2, 3), Region 3 had a positive measure, in Figure 4, where we plot μ on the horizontal axis, Region 3 collapses to a single value of μ.[21]

It is important to notice that a monetary equilibrium cannot be supported for very large

[21] This is an immediate consequence of the indeterminacy of money demand, and it can be seen more clearly by inspection of Figure 3: it follows from that graph that the buyer's asset holdings will bring her in Region 4 if and only if $\varphi/[\beta\hat{\varphi}(1-l)] > 1/(1-l)$, which is equivalent to $\varphi > \beta\hat{\varphi}$, and reduces to $\mu > \beta - 1$ in steady state equilibrium. Similarly, the buyer's asset holdings will bring her in Region 2 if and only if $\varphi < \beta\hat{\varphi}$, which is equivalent to $\mu < \beta - 1$.

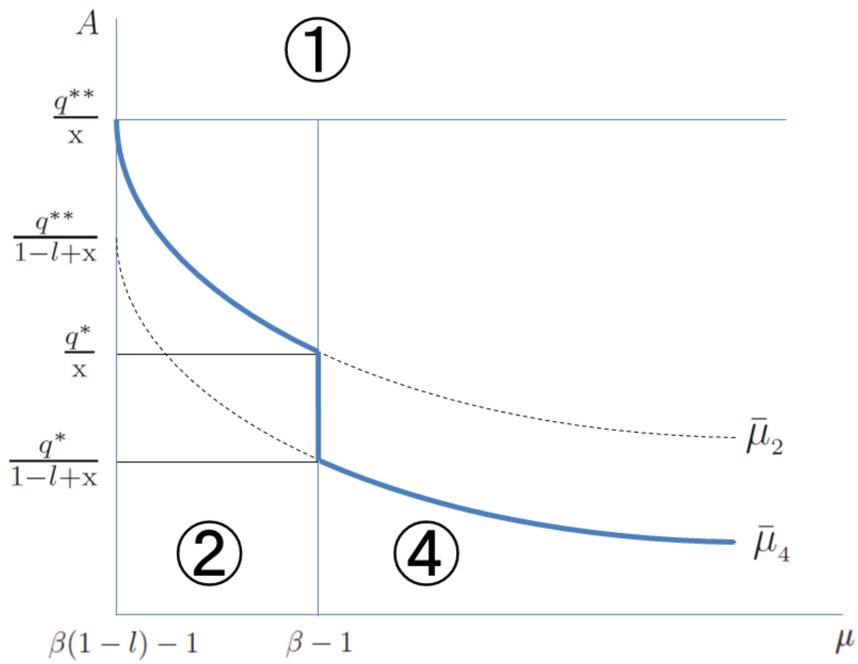

Figure 4: Regions of equilibrium as functions of the parameters A and μ.

values of μ. To see this, recall from Definition 1 that q is increasing in z, which, in turn, is decreasing in μ. Hence, there exists a critical value $\bar{\mu}$, such that setting $\mu > \bar{\mu}$ would lead to an equilibrium q which is lower than the value that q would obtain in a world with no money (i.e., in a world where the buyer purchases q only by using the asset as collateral). Clearly, an optimally behaving buyer would never choose to pay a positive price ($\varphi > 0$) in order to acquire an object that ends up hurting her in equilibrium. That is to say, for any $\mu > \bar{\mu}$, the equilibrium is non-monetary ($z = 0$), and the various equilibrium objects are functions of A only.

The level of μ above which the monetary equilibrium collapses is depicted in Figure 4 by the bold blue piece-wise curve. For $\mu < \beta - 1$ ($\mu > \beta - 1$), the relevant part of the curve coincides with $\bar{\mu}_2$ ($\bar{\mu}_4$), since for this range of μ's equilibrium lies in Region 2 (Region 4). Both $\bar{\mu}_2$ and $\bar{\mu}_4$ are decreasing in A, since the larger A is, the better buyers can do without money (in terms of purchasing q), and the less tolerant they will be to inflation. As we show in the appendix, the critical values $\bar{\mu}_i = \bar{\mu}_i(A)$, $i = 2, 4$, are given by

$$\begin{aligned}\bar{\mu}_2(A) &= \beta u'(xA) - 1,\\ \bar{\mu}_4(A) &= \beta u'((1-l+x)A) - 1.\end{aligned}$$

We are now ready to state the main results of the paper. Proposition 1 describes the equilibrium asset price and how it is affected by inflation.

Proposition 1. *a) Suppose $A \geq q^{**}/x$. Then, the asset price equals the fundamental value, i.e., $\psi = \beta(1-l)$, and it is not affected by μ.*

b) Suppose $A \in (q^*/x, q^{**}/x)$. Then, for any $\mu < \bar{\mu}_2(A) < \beta - 1$, a monetary equilibrium exists in which the asset price is given by $\psi(\mu) = \psi_2(\mu) = \beta(1-l) + x(1+\mu)$. For any $\mu \geq \bar{\mu}_2(A)$ the equilibrium is non-monetary, and the asset price is given by $\psi(\mu) = \psi_2(\bar{\mu}_2(A))$.

c) Suppose $A \in [q^*/(1-l+x), q^*/x]$. Then, for any $\mu < \beta - 1$, a monetary equilibrium exists in which the asset price is given by $\psi(\mu) = \psi_2(\mu) = \beta(1-l)+x(1+\mu)$. For any $\mu \geq \beta - 1$ the equilibrium is non-monetary, and the asset price is given by $\psi(\mu) = \psi_2(\bar{\mu}_2(A))$, evaluated at $A = q^*/x$.

d) Suppose $A < q^*/(1-l+x)$. Then, for any $\mu < \bar{\mu}_4(A)$, a monetary equilibrium exists. Within this range of μ's, if $\mu < \beta - 1$, we have $\psi(\mu) = \psi_2(\mu) = \beta(1-l) + x(1+\mu)$, and if $\mu \in (\beta - 1, \bar{\mu}_4(A))$, we have $\psi(\mu) = \psi_4(\mu) = (1-l+x)(1+\mu)$. For any $\mu \geq \bar{\mu}_4$, the equilibrium is non-monetary, and the asset price is given by $\psi(\mu) = \psi_4(\bar{\mu}_4(A))$.

In summary, with the exeption of case (a), the asset price exceeds the fundamental value ($\beta(1-l)$), and it is a strictly increasing function of μ in all monetary equilibria.

Proof. The proof follows directly from Lemma 3 and inspection of Figure 4, which identifies the relevant region of equilibrium for any (A, μ). Consider case (d), which is the richest because, depending on the value of μ, equilibrium can lie in either Region 2 or 4. Existence of monetary equilibrium requires that μ does not exceed a certain bound. Since here $A < q^*/(1-l+x)$, this bound is given by $\bar{\mu}_4(A) > \beta - 1$. Assuming $\mu < \bar{\mu}_4(A)$, the equilibrium (which is monetary) lies in Region 2, if $\mu < \beta - 1$, and in Region 4, if $\mu \in (\beta - 1, \bar{\mu}_4(A))$. To calculate the asset price focus first on Region 2. Imposing the equilibrium conditions $\hat{\varphi}\hat{m} = z$, $\hat{a} = A$, and $\varphi/\hat{\varphi} = 1 + \mu$, in the relevant first-order conditions (i.e., the equations (11),(12) set equal to zero) implies

$$\begin{aligned} 1 + \mu =& \beta u'(z + xA), \\ \psi =& \beta[u'(z + xA)x + 1 - l]. \end{aligned}$$

Solving this system of equations delivers a formula for the asset price as a function of μ. Since we are in the case where equilibrium lies in Region 2, we term it $\psi_2(\mu) = \beta(1-l) + x(1+\mu)$.

To finish the proof for case (d), consider an equilibrium in Region 4, i.e., let $\mu \in (\beta - 1, \bar{\mu}_4(A))$. Now the relevant first-order conditions are given by equations (15),(16) set equal to zero. Imposing the same equilibrium conditions as above implies

$$\begin{aligned} 1 + \mu =& \beta u'(z + (1-l+x)A), \\ \psi =& \beta u'(z + (1-l+x)A)(1-l+x). \end{aligned}$$

Solving this system of equations delivers a formula for the asset price as a function of μ, specifically, $\psi_4(\mu) = (1-l+x)(1+\mu)$, where the index 4 refers to the relevant region of equilibrium.

The proof for part (a) is trivial. The proofs for parts (b),(c) follow similar steps. □

Proposition 1 is very intuitive. Given the competitive nature of the CM, and the fact that the supply of the asset is fixed at A, we know that the equilibrium price of the asset will reflect the value that the buyer assigns to the "last" (marginal) unit. If $A \geq q^{**}/x$ (equilibrium lies in Region 1), the asset supply is so plentiful that it allows the buyer to purchase the amount q^{**} in the LW market without using any money or credit. As a result, the buyer values the marginal unit of the asset only as a store of value, which is equivalent to saying that the only possible equilibrium price is the fundamental value $\psi = \beta(1 - l)$.

Consider now the more interesting case where $A < q^{**}/x$. A monetary equilibrium exists if and only (A, μ) lies on the left of the blue piece-wise curve depicted in Figure 4. If a monetary equilibrium exists and $\mu < \beta - 1$, the asset price is given by $\psi_2(\mu) = \beta(1 - l) + x(1 + \mu)$. On the other hand, if a monetary equilibrium exists and $\mu > \beta - 1$, we have $\psi = \psi_4(\mu) = (1 - l + x)(1 + \mu)$. In both cases, ψ exceeds the fundamental value: buyers are willing to pay a premium, a liquidity premium, because the marginal unit of the asset helps them increase consumption in the LW market. Also, in both cases, the asset price is increasing in μ, since the asset serves effectively as a substitute to money: as inflation rises, buyers increase their demand for the real asset, which, in turn, leads to a higher price. Notice that the slope of $\psi_4(\mu)$ is greater than the slope of $\psi_2(\mu)$, because in Region 4 an additional unit of the asset helps the buyer achieve an even greater increase in q through its ability to relax the collateral constraint. This is not the case in Region 2, where the buyer has already reduced the value of her loan to zero.

It is important to notice that although money and the real asset are substitutes, they are not *perfect substitutes*: money is used in a *quid pro quo* fashion, i.e., it is exchanged for the good, while the asset serves as collateral that helps the buyer issue a secured loan, and it will end up in the seller's hands only in the case of default. This explains why in this analysis the rate of return on money and the asset are positively related, but not equal, as is the case in papers like Geromichalos *et al* (2007).[22]

The main results of Proposition 1 are summarized in Figure 5. The figure has been plotted for $A < q^*/(1 - l + x)$, which is consistent with equilibrium in both Regions 2 and 4. Notice that $\psi_2(\beta - 1) = \psi_4(\beta - 1) = \beta(1 - l + x)$, so that $\psi(\mu)$ is continuous at $\beta - 1$.

Proposition 1 can be exploited in order to study the effect of the secondary asset market frictions on the (primary) price of the asset, ψ. As we have already explained, the price ψ reflects the buyer's marginal value of the asset. Interestingly, although the buyers do not even participate in the OTC market, the asset price ψ is critically affected by the terms α_s (the probability with which a seller meets an investor in the OTC market) and λ (the bargaining power of the

[22] The net rate of return on money is $(\hat{\varphi} - \varphi)/\varphi = 1/(1 + \mu) - 1$, and the net rate of return on the asset is $(1 - \psi)/\psi$. Equality of these rates would require $\psi = 1 + \mu$, which is clearly not the case here. However, the two rates are positively related, in the sense that an increase in μ decreases (directly) the rate of return on money, but it also decreases (indirectly) the rate of return on the asset by increasing its price.

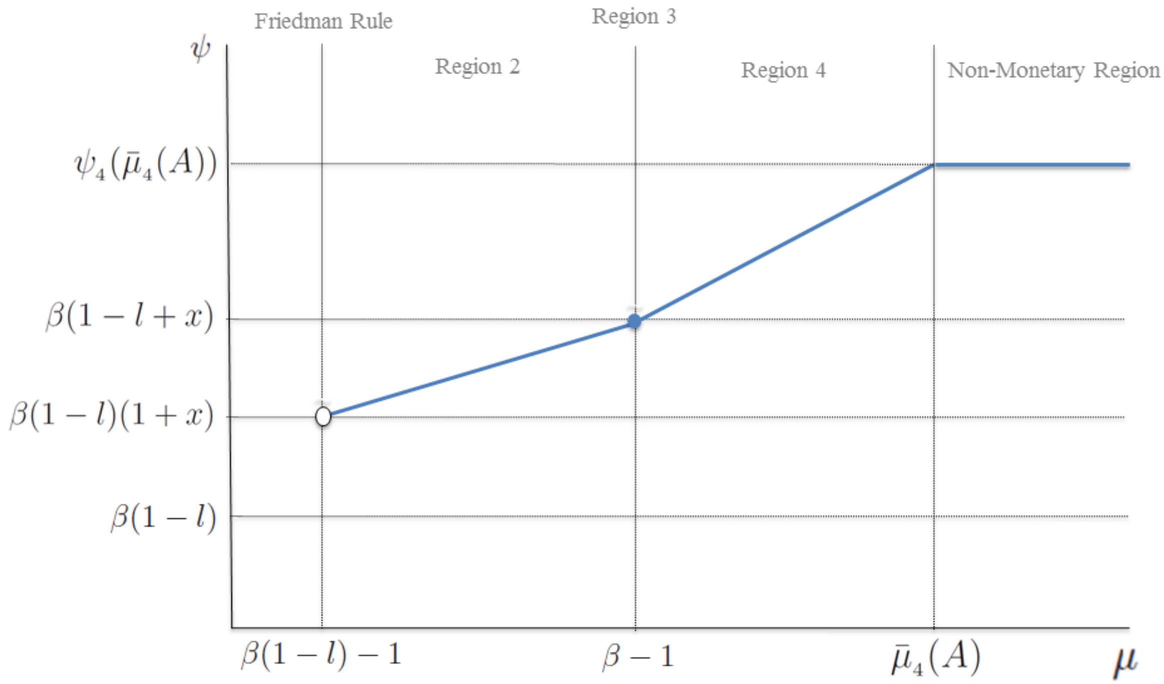

Figure 5: Effect of money growth on equilibrium asset price with $A < \bar{A}$

seller in OTC meetings). To illustrate the nature of this dependence in more detail, assume that $A < q^*/(1-l+x)$ and focus on monetary equilibria (i.e., let $\mu < \bar{\mu}_4$). It can be easily verified that, regardless of the relevant region of equilibrium, we have

$$\frac{\partial \psi}{\partial \alpha_s} = (1+\mu)l\lambda\delta > 0,$$
$$\frac{\partial \psi}{\partial \lambda} = (1+\mu)l\alpha_s\delta > 0.$$

Hence, a higher degree of liquidity in the secondary asset market (interpreted as a higher value of α_s) and/or a higher bargaining power λ will convince a seller to accept the asset as collateral, even though she might not have a high valuation for the asset herself. The buyer (who has rational expectations) realizes that, and she is willing to purchase the asset at a premium, which is increasing in both α_s and λ. Notice that the asset price carries a liquidity premium (and the asset serves as collateral in the LW market) even if the seller has no value for the asset whatsoever, i.e., even if $\delta = 1$: in this case the lowest possible value of $\psi(\mu)$, i.e., the one that ψ attains as $\mu \to \beta(1-l)-1$, is given by $\beta(1-l)(1+l\alpha_s\lambda)$, which clearly exceeds the fundamental value. Since $\psi(\mu) > \beta(1-l)$ even at the (modified) Friedman rule, clearly the same will be true for any other μ, since from Proposition 1 we know that $\psi'(\mu) \geq 0$ (strictly for $\mu < \bar{\mu}_4$).

The next proposition describes the equilibrium production in the LW market, which, in this model is a sufficient statistic for equilibrium welfare.

Proposition 2. *a) Suppose $A \geq q^{**}/x$. Then, $q = q^{**}$.*

b) Suppose $A \in (q^*/x, q^{**}/x)$. Equilibrium always lies in Region 2, and the LW market production is given by $q_2 = \{q : 1 + \min\{\mu, \bar{\mu}_2(A)\} = \beta u'(q)\} > q^*$.

c) Suppose $A \in [q^*/(1-l+x), q^*/x]$. If $\mu < \beta - 1$, equilibrium lies in Region 2, and the LW market production is given by q_2 defined in part (b). If $\mu = \beta - 1$, we have $q = q^*$.

d) Suppose $A < q^*/(1-l+x)$. If $\mu < \beta - 1$, equilibrium lies in Region 2, and the LW market production is given by q_2 defined in part (b). If $\mu = \beta - 1$, we have $q = q^*$. Finally, if $\mu > \beta - 1$, equilibrium lies in Region 4, and the LW market production is given by $q_4 = \{q : 1 + \min\{\mu, \bar{\mu}_4(A)\} = \beta u'(q)\} < q^*$.

Proof. The proof also follows directly from the optimal behavior of the buyer (Lemma 3) and inspection of Figure 4. For any (A, μ) that lies on the left of the blue piece-wise curve, we have a monetary equilibrium. In this case, the equilibrium LW production solves $\{1 + \mu = \beta u'(q)\}$, regardless of the relevant region. This follows directly from the first-order conditions (11),(15). Although the equilibrium q is given (implicitly) by the same formula in both Regions 2 and 4, these cases are qualitatively different: if $\mu = \beta - 1 + \epsilon$, for a small but positive ϵ, equilibrium lies in Region 4, and $q = z + (1-l+x)A$, i.e., the buyer uses all her assets as collateral. On the other hand, if $\mu = \beta - 1 - \epsilon$, equilibrium lies in Region 2, and $q = z + xA$, i.e., the buyer does not obtain any loan. When μ is exactly equal to $\beta - 1$, which is equivalent to saying that the equilibrium is in Region 3, naturally we have $q = q^*$, but the level of z is indeterminate.

If μ is so large that equilibrium becomes non-monetary, then the equilibrium quantity solves $\{1 + \bar{\mu}_i(A) = \beta u'(q)\}$, where $i = 2, 4$ captures the relevant region in Figure 4. □

The function $q(m)$ is depicted in Figure 6, for $A < q^*/(1-l+x)$, which is the most interesting case. The equilibrium quantity is decreasing in μ throughout the admissible range of monetary policies, and constant for $\mu > \bar{\mu}_4$. The adjusted Friedman rule, $\mu = \beta(1-l) - 1$, is optimal, and it leads to the first best equilibrium quantity q^{**}. Notice that $q(m)$ is not only continuous at $\mu = \beta - 1$ (this was also the case for $\psi(\mu)$), but also differentiable at that point (this was not the case for $\psi(\mu)$, which exhibited a kink at $\mu = \beta - 1$). This follows from the fact that $q(m) = \{q : 1 + \mu = \beta u'(q)\}$ regardless of which region the equilibrium lies in.

In the last part of this section, we determine the "haircut" that is applied to the collateral asset and study how it is affected by inflation. Following the standard approach in finance, we define the haircut as the percentage that is subtracted from the value of an asset that is used as collateral. Equivalently, the haircut can be defined as 1 minus the Loan to Value (LTV) ratio, i.e., $1 - b/(\psi A)$ in our model. The following proposition states the relevant result.

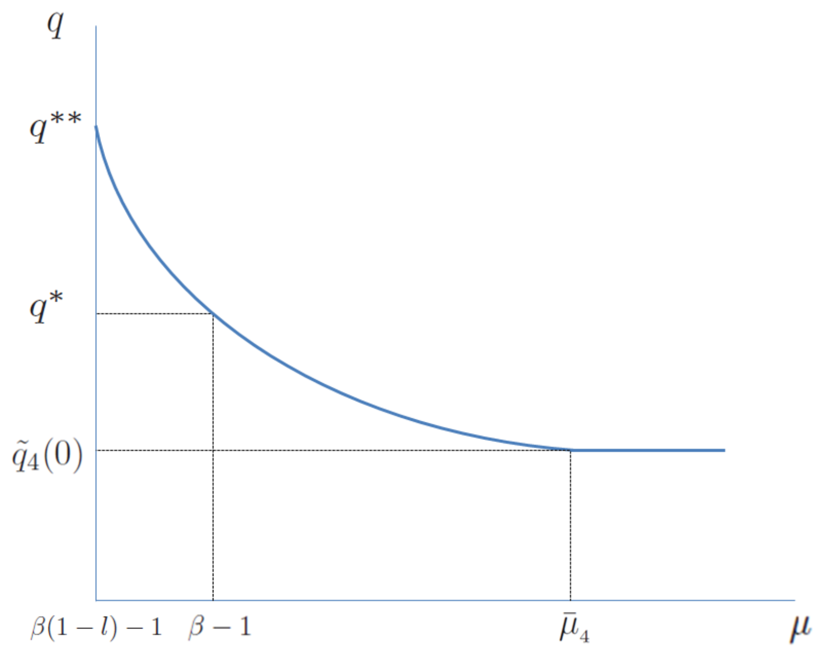

Figure 6: Effect of money growth on equilibrium quantity for $A < q^*/(1-l+x)$. The term $\tilde{q}_4(0)$ is understood to be the function $\tilde{q}_4(z)$ (from Definition 1) evaluated at $z = 0$.

Proposition 3. *Define the haircut of the asset as $\Xi \equiv 1 - b/(\psi A)$, and focus on equilibria that lie in Regions 3 and 4, since in Regions 1 and 2 the buyer does not obtain a loan. Then, the haircut is increasing in both μ and α_s.*

Proof. See the appendix. □

In order to understand the results stated in Proposition 3, focus on equilibria that lie in Region 4 (the intuition for Region 3 is similar, although the proof is slightly more complicated; see the appendix). From the definition of a steady state equilibrium, we know that in Region 4 we have $b = A$, i.e., the buyer pledges all of her asset as collateral in order to obtain the maximum possible loan. Then, the definition of Ξ implies that $\Xi = 1 - 1/\psi_4$, where ψ_4 is defined in Proposition 1. In words, since the loan is equal to the total amount of assets that the buyer carries (and equal to A in equilibrium), the haircut is simply given by 1 minus the inverse of the asset price, and, hence, Ξ and ψ will move in the same direction. Consequently, anything that makes the asset more valuable, will tend to decrease the LTV ratio and increase the haircut. Since, as we have seen, $\partial\psi_4/\partial\mu, \partial\psi_4/\partial\alpha_s > 0$, it also follows that $\partial\Xi/\partial\mu, \partial\Xi/\partial\alpha_s > 0$. For instance, an increase in μ makes carrying money more expensive, and induces the buyer to rely more heavily on her asset, which she can use as collateral. For any given A the buyer obtains the same amount of loan ($b = A$), but the value of the collateral that she has to pledge to the seller in order to obtain this loan (i.e., the haircut) has now gone up, because ψ has gone up.

Many of the results of our paper (e.g., the positive relationship between asset prices and inflation and the fact that asset prices include liquidity premia) are supported by findings in the existing empirical literature. This is not true with respect to the result reported in Proposition 3, because not much empirical work has been done on the link between haircuts and asset market liquidity. To that end, we present some evidence which is directly in support of our theoretical findings. Although our model describes the pricing properties of any asset that can serve as collateral, an example that fits our story well is houses that agents use as collateral in order to obtain loans.[23] Tables 1, 2, and 3 summarize the values of the Loan-to-Value (LTV) ratio on outstanding loans and the number of housing transactions (which serves as a proxy for market liquidity) for the US, Japan, and Korea, respectively.[24] All three tables reveal that there is a clear negative correlation between the LTV ratio and the volume of transactions in the housing market. In particular, the numbers in parenthesis in these tables represent the growth rates of the LTV ratio and the housing trade volume, respectively. For instance, in Table 1, with the exception of 2009 and 2011, every year in which there was a decrease (increase) in the housing trade volume was characterized by an increase (decrease) in the LTV ratio. The same observation holds true in Table 2 (Japan) and in Table 3 (Korea), with the exception of 2011. Given that the LTV ratio is defined as 1 minus the haircut, one can argue that the data provides clear support to the result stated in Proposition 3.

Table 1: LTV ratio and Housing Transactions in the United States

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014
LTV Ratio[1]	57	63	72	77	78	80	75	69	66
	(1.8)	(10.5)	(14.3)	(6.9)	(1.3)	(2.6)	(-6.3)	(-8.0)	(-4.3)
Housing Transactions[2]	6477	5030	4110	4340	4190	4260	4660	5090	4940
	(-8.5)	(-22.3)	(-18.3)	(5.6)	(-3.5)	(1.7)	(9.4)	(9.2)	(-2.9)

Notes: 1) Weighted Average Estimated Loan-to-Value Ratio of Our Single-family Credit Guarantee Portfolio Adjusted to Reflect Current Market Values. 2) Existing-home sales are based on closing transactions of single-family, townhomes, condominiums and cooperative homes. 3) The numbers in parenthesis indicate year on year growth rates. 4) Housing Transactions are in thousands. Source: Freddie Mac Update March 2015, and National Association of Realtors through FRED.

[23] The example of houses being used as collateral fits well with our story in a *conceptual* sense. Recall that our story entails agents who use assets as collateral but these assets might not be valued *per se* by lenders. However, lenders still accept them as collateral, because they understand that the borrowers have an incentive to repay their debt and keep the asset, and because they know that even if the borrower has no choice but to default, they could sell off the collateral in a secondary market. These are all features that also characterize the types of transactions that take place in our real-world example.

[24] Using the trade volume in an asset market as a proxy for market liquidity is a standard approach in the empirical finance literature; see for example Ashcraft and Duffie (2007) and Gürkaynak, Sack, and Wright (2010).

Table 2: LTV ratio and Housing Transactions in Japan

Year	1994	1999	2004	2009
LTV Ratio	23.0	36.8	46.8	51.0
		(60.0)	(27.2)	(9.0)
Housing Transactions[1]	9637	9074	8152	7041
		(-5.8)	(-10.2)	(-13.6)

Notes: 1) Housing Transactions are the cumulative total for the past 5 years in thousands. 2) The numbers in parenthesis indicate period on period growth rates. Source: Bank of Japan, and Ministry of Land, Infrastructure, Transport and Tourism.

Table 3: LTV ratio and Housing Transactions in South Korea

Year	2008	2009	2010	2011	2012
LTV Ratio	47.1	47.5	48.0	48.4	50.5
	(-1.9)	(0.8)	(1.0)	(0.2)	(4.9)
Housing Transactions	894	870	800	981	735
	(3.0)	(-2.7)	(-8.4)	(20.4)	(-28.9)

Notes: 1) The LTV ratio in 2012 is measured at the end of June in 2012. 2) The numbers in parenthesis indicate year on year growth rates. 3) Housing Transactions are in thousands. Source: Korean Financial Supervisory Service, and Ministry of Land, Infrastructure and Transport.

5 Conclusion

We develop a model where part of the economic activity takes place in markets where certain frictions, such as anonymity and limited commitment hinder unsecured credit. We show that the property of assets to serve as collateral in these markets crucially affects their equilibrium price, which exceeds the fundamental value, i.e., it contains a liquidity premium. This premium is increasing in inflation, because inflation raises the opportunity cost of holding money, and the asset serves effectively as a substitute to money. However, inflation hurts the economy's welfare. Consistent with anecdotal evidence, we show that the price of the asset is positively linked to the liquidity of the secondary asset market. This is true even though the original buyers of the asset (i.e., agents who purchase the asset in order to use it as collateral) do not even participate in the OTC market. Furthermore, we show that sellers of rationally choose to accept the collateral and extend loans to buyers, even when their personal valuation for the collateral asset is very low or zero. Finally, both a higher level of inflation and a higher probability of trade in the OTC market lead to an increase of the haircut applied to the collateral asset.

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A Appendix

Proof. Proof of Lemma 2.

We define the Langrangian function for the LW bargaining problem as

$$\begin{aligned} L &= u(q) - (1-l)\varphi d - (1-l)b - \lambda_1(d-m) - \lambda_2(-b) - \lambda_3(b-a) \\ &- \lambda_4[q - \varphi d - (1-l)b - l(1-\delta+\alpha_s\lambda\delta)a], \end{aligned}$$

where λ_1, λ_2, λ_3, and λ_4 are Lagrange multipliers. The FOCs are

$$\{q\} \; : \; u'(q) - \lambda_4 = 0, \tag{a.1}$$

$$\{d\} \; : \; -(1-l)\varphi - \lambda_1 + \lambda_4\varphi = 0, \tag{a.2}$$

$$\{b\} \; : \; -(1-l) + \lambda_2 - \lambda_3 + \lambda_4(1-l) = 0, \tag{a.3}$$

$$\{\lambda_1\} \; : \; d - m \leq 0, \quad \lambda_1 \geq 0, \tag{a.4}$$

$$\{\lambda_2\} \; : \; -b \leq 0, \qquad \lambda_2 \geq 0, \tag{a.5}$$

$$\{\lambda_3\} \; : \; b - a \leq 0, \quad \lambda_3 \geq 0, \tag{a.6}$$

$$\{\lambda_4\} \; : \; q - \varphi d - (1-l)b - l(1-\delta+\alpha_s\lambda\delta)a = 0. \tag{a.7}$$

Case 1: $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = 0$ ($d < m$, $0 < b < a$)
Equations (a.1) and (a.2) imply $u'(q) = 1 - l$. On the other hand, equations (a.1) and (a.3) imply $u'(q) = 1$. These contradict each other.
Case 2: $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 > 0$ ($d < m$, $0 < b = a$)
Equations (a.1) and (a.2) imply $u'(q) = 1 - l$. On the other hand, equations (a.1) and (a.3) imply $\lambda_3 = (1-l)[u'(q) - 1]$. Since $\lambda_3 > 0$, this implies $u'(q) > 1$. Again, these contradict each other.
Case 3: $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 = 0$ ($d < m$, $0 = b < a$)
Equations (a.1) and (a.2) imply $u'(q) = 1 - l$. Hence, $q = q^{**}$. Equations (a.1) and (a.3) imply $\lambda_2 = (1-l)[1 - u'(q^{**})]$, which is consistent with $\lambda_2 > 0$. Equation (a.5) implies $b = 0$. Then, from equation (a.7), we get $d = [q^{**} - l(1-\delta+\alpha_s\lambda\delta)a]/\varphi$, i.e., $d = (q^{**} - xa)/\varphi$. To sum up, in Case 3 the solution is $q = q^{**}$, $d = (q^{**} - xa)/\varphi$, and $b = 0$. Notice that in this case we have

$\pi(m,a) > q^{**}$, which is consistent with the fact that $d < m$.

Case 4: $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 > 0$ ($d < m$, $0 = b = a$)

This case is similar to Case 3, with the exception that here $a = 0$. It is easy to check that the solution is $q = q^{**}$, $d = q^{**}/\varphi$, and $b = 0$. Since here $d < m$ and $a = 0$, we have $\pi(m,0) > q^{**}$.

Case 5: $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 = 0$ ($d = m$, $0 < b < a$)

Equations (a.1) and (a.3) imply $u'(q) = 1$, and so $q = q^*$. Notice that $q^{**} > q^*$ because $u'' < 0$. Equations (a.1) and (a.2) imply $\lambda_1 = \varphi[u'(q^*) - (1-l)]$, which is consistent with $\lambda_1 > 0$. Equation (a.4) implies $d = m$. Then, from equation (a.7), we get $b = \big[q^* - \varphi m - l(1-\delta+\alpha_s\lambda\delta)a\big]/(1-l)$, i.e., $b = \big[q^* - \pi(m,a)\big]/(1-l)$. To sum up, the solution is $q = q^*$, $d = m$, and $b = [q^* - \pi(m,a)]/(1-l)$. Since here $0 < b < a$, we have $q^* - (1-l)a < \pi(m,a) < q^*$.

Case 6: $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 > 0$ ($d = m$, $0 < b = a$)

Equations (a.1) and (a.3) imply $u'(q) > 1$. Equations (a.1) and (a.2) imply $u'(q) > 1-l$. Therefore, the solution of q must satisfy $q < q^*$. Equation (a.4) implies $d = m$. Equation (a.6) implies $b = a$. Then, from equation (a.7), we get $q = \varphi m + [1 - l + l(1-\delta+\alpha_s\lambda\delta)]a$, i.e., $q = \pi(m,a) + (1-l)a$. To sum up, in this case the solution is $q = \pi(m,a) + (1-l)a$, $d = m$, and $b = a$. Notice that since here $q < q^*$, we have $\pi(m,a) < q^* - (1-l)a$.

Case 7: $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 = 0$ ($d = m$, $0 = b < a$)

Equations (a.1) and (a.3) imply $u'(q) < 1$. Equations (a.1) and (a.2) imply $u'(q) > 1 - l$. Thus, $1 - l < u'(q) < 1$, which means that $q^* < q < q^{**}$. Equation (a.4) implies $d = m$. Equation (a.5) implies $b = 0$. Then, from equation (a.7), we get $q = \varphi m + l(1-\delta+\alpha_s\lambda\delta)a$, i.e., $q = \pi(m,a)$. To sum up, in this case the solution is $q = \pi(m,a)$, $d = m$, and $b = 0$. Since here $q^* < q < q^{**}$, we have $q^* < \pi(m,a) < q^{**}$.

Case 8: $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 > 0$ ($d = m$, $0 = b = a$)

Equations (a.1) and (a.2) imply $u'(q) > 1 - l$. Thus, $q < q^{**}$. Equation (a.4) implies $d = m$. Equation (a.5) implies $b = 0$. Then, from equation (a.7), we get $q = \varphi m$. Summing up, we have $q = \varphi m$, $d = m$, and $b = 0$. Since here $q < q^{**}$, we have $q^{**} > \pi(m,0)$. □

Proof. Proof of Lemma 3.

As a first step, we highlight some important properties of $J : \mathbf{R}^2_+ \to \mathbf{R}$. First, this function is continuous throughout its domain. To see why this is the case, recall that the solution to the LW market bargaining problem is continuous, and so is u. Hence, J is also continuous.

Second, the function J is differentiable within each of the four regions defined in Lemma 2. To see this point, notice that the solution to the bargaining problem is differentiable within each of the four regions defined in Lemma 2, and u is also (twice continuously) differentiable. Therefore, J is differentiable within each of the four regions.

Third, J is weakly concave in both arguments everywhere. To see this point, recall that J is continuous everywhere and differentiable within each region, thus J_1 and J_2 are well-defined within each region. Moreover, J_1 and J_2 are constant in Regions 1 and 3, and strictly decreasing in Regions 2 and 4 in $\hat{m}$ and $\hat{a}$ because $u'' < 0$. Thus, J is weakly concave everywhere.

Given the propoerties of J, we have the following:

a) Since J is weakly concave and differentiable (with the exception of the boundary points), the optimal choice ($\hat{m}$, $\hat{a}$) in each region satisfies $\nabla J(\hat{m}, \hat{a}) = \mathbf{0}$.

b) If the asset price is equal to the fundamental value, i.e., $\psi = \beta(1-l)$, Regions 2, 3, and 4 are ruled out, because $\psi = \beta(1-l)$ implies that $J_2^1 = 0$, while J_2^2, J_2^3 and $J_2^4 > 0$ for any bundle ($\hat{m}$, $\hat{a}$). There is no reason to choose less $\hat{a}$ than q^{**}/x in this case. On the other hand, the fact that $J_1^1 < 0$ in the case where $\varphi > \beta\hat{\varphi}(1-l)$, i.e., the cost of holding money is positive means that $\hat{m} = 0$ because a buyer can purchase the amount q^{**} without using any money. Hence, any bundle ($\hat{m}$, $\hat{a}$) with $\hat{m} = 0$ and $\hat{a} \geq q^{**}/x$ is optimal.

c) When $\psi > \beta(1-l)$, any choice within Region 1 is ruled out because $J_2^1 < 0$. In Regions 2 and 4, J_1 and J_2 are strictly decreasing in $\hat{m}$ and $\hat{a}$, therefore the optimal choice ($\hat{m}$, $\hat{a}$) satisfying $J_1 = 0$ and $J_2 = 0$ is unique (given that $\varphi > \beta\hat{\varphi}(1-l)$ and $\psi > \beta(1-l)$). In Region 3, since any bundle ($\hat{m}$, $\hat{a}$) satisfies $J_1^3 = 0$ and $J_2^3 = 0$, the optimal choice is indeterminate. □

Proof. Proof of Lemma 4.

From Lemma 1, it is straightforward to show that χ and c exist and are unique, given A, in the equilibrium. From Lemma 2 and 3, we have the following equations on each region in the steady state where $\varphi/\hat{\varphi} = 1+\mu$.

$1+\mu = \beta(1-l)$, $\psi = \beta(1-l)$, $q = q^{**}$ and $b = 0$, in Region 1,

$1+\mu = \beta u'(\tilde{q}_2(z))$, $\psi = \beta\big[u'(\tilde{q}_2(z))x + 1 - l\big]$ and $b = 0$, in Region 2,

$1+\mu = \beta$, $\psi = \beta(1-l+x)$, $q = q^{*}$, and $b = \tilde{b}(z)$, in Region 3,

$1+\mu = \beta u'(\tilde{q}_4(z))$, $\psi = \beta u'(\tilde{q}_4(z))(1-l+x)$ and $b = A$, in Region 4.

Clearly, ψ, z, q, and b exist and are unique in Region 1. In Regions 2 and 4, μ uniquely pins down z, q and ψ. In addition, b is unique in all regions. Hence, equilibrium exists and is unique in Regions 2 and 4. In Region 3, z can obtain any value in the interval of $[q^* - (1-l+x)A, q^* - xA]$, even though ψ and q are unique. It follows that b is also indeterminate because it is a function of z. Hence, they all exist, but z and b are not unique in Region 3. □

Proof. Derivation of expressions $\bar{\mu}_i(A)$, $i = 2, 4$, in Section 4.2.

Consider Region 2. For any given A, the term $\bar{\mu}_2$ indicates the level of inflation that would lead to the same q as in the case where $z = 0$. The latter is simply given by $q_{2,n} = xA$ (the second index, n, stands for "non-monetary" equilibrium). To identify the relationship between μ and q in an equilibrium where money is valued, set the derivative in (11) equal to zero, and evaluate it in steady state. This yields $1+\mu = \beta u'(q_{2,m})$, where $q_{2,m} = z + xA$ (in the same spirit as above, the second index, m, stands for "monetary" equilibrium). Hence, $\bar{\mu}_2$ solves $\bar{\mu}_2 = \{\mu : q_{2,m} = q_{2,n}\}$, which implies that $\bar{\mu}_2 = \beta u'(xA) - 1$. The derivation of $\bar{\mu}_4$ follows identical steps. □

Proof. Proof of Proposition 3.

Consider first equilibria in Region 4. From the definition of steady state equilibrium we know that in this region $b = A$. Also, from Proposition 1 we know that $\psi = \psi_4(\mu)$. Hence, in this region the haircut is given by $\Xi = 1 - 1/\psi_4$. Consequently, we have

$$\begin{aligned}\frac{\partial \Xi}{\partial \mu} =& \frac{\partial \Xi}{\partial \psi_4}\frac{\partial \psi_4}{\partial \mu} = \frac{1}{\psi_4^2}\frac{\partial \psi_4}{\partial \mu}, \\ \frac{\partial \Xi}{\partial \alpha_s} =& \frac{\partial H}{\partial \psi_4}\frac{\partial \psi_4}{\partial \alpha_s} = \frac{1}{\psi_4^2}\frac{\partial \psi_4}{\partial \alpha_s},\end{aligned}$$

and since we have established that ψ_4 is increasing in both μ and α_s, the same is true about Ξ.

In Region 3, it is impossible to obtain an expression for $\partial\Xi/\partial\mu$, since being in this region requires $\mu = \beta - 1$. However, we can study how Ξ depends on α_s. To that end, recall from the definition of equilibrium that in Region 3 we have $b = \tilde{b}(z) = (1-l)^{-1}(q^* - z - xA)$. Also, from the discussion that follows Proposition 1, recall that the asset price is given by $\psi = \psi_4(\beta - 1) = \beta(1 - l + x)$. Hence, in Region 3 the haircut is given by

$$\Xi = 1 - \frac{q^* - z - xA}{(1-l)\beta(1-l+x)A}$$

. It is now straightforward to verify that

$$\frac{\partial \Xi}{\partial \alpha_s} = \frac{\partial \Xi}{\partial x}\frac{\partial x}{\partial \alpha_s} = \frac{A(1-l+x) + \tilde{b}(z)(1-l)}{(1-l)\beta(1-l+x)^2 A}\, l\lambda\delta,$$

which is clearly strictly positive. □