Online Appendix to "Unemployment Insurance and Optimal Taxation in a Search Model of the Labor Market"

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In this document, I analyze the case of a "small market", i.e. a market with few strategic firms who realize that their actions can affect market outcomes. I provide closed form solutions for the equilibrium wage and profits under the three systems that are available to raise UI funds.

Consider first a market with m firms and n risk-neutral workers. Since, the goal is to construct symmetric equilibria, I focus on the behavior of the typical firm j, and I assume that all other firms post the same wage $\tilde{w}$. Firm j chooses its wage w^j, realizing that in the second stage of the game workers will observe $(w^j, \tilde{w})$ (and the z announced by the government), and they will determine the probabilities with which they apply to each firm, so that they are indifferent among all firms. Let θ denote the probability with which the typical worker applies to firm j, so that $\tilde{\theta} = (1-\theta)/(m-1)$ stands for the probability with which she applies to any of the other $m-1$ firms. Then, under System $i = \{A, B\}$, firm j wishes to solve the following maximization problem:

$$\max_{w^j} \left\{ \left[1-(1-\theta)^n\right](y-w^j) - \tau^i \right\}$$
$$s.t. \; \frac{1-(1-\theta)^n}{n\theta} w^j + \left[1 - \frac{1-(1-\theta)^n}{n\theta}\right] z = \frac{1-\left(1-\tilde{\theta}\right)^n}{n\tilde{\theta}} \tilde{w} + \left[1 - \frac{1-\left(1-\tilde{\theta}\right)^n}{n\tilde{\theta}}\right] z,$$

where τ^i represents the tax payment under System $i = \{A, B\}$, given by

$$\begin{aligned}
\tau^A &= \frac{1}{m}\left\{ n\theta - 1 + (1-\theta)^n + (m-1)\left[n\tilde{\theta} - 1 + \left(1-\tilde{\theta}\right)^n\right]\right\} z, \\
\tau^B &= \left[n\theta - 1 + (1-\theta)^n\right] z.
\end{aligned}$$

A few comments are in order. The term $1-(1-\theta)^n$ represents the probability with which a firm receives one or more applications. Therefore, when a worker applies to a firm which is chosen by other workers with probability θ, the probability with which she will get the job is given by $[1-(1-\theta)^n]/(n\theta)$. The tax payment of the typical firm depends on the system under consideration. Under System A, the firm pays a fraction $1/m$ of the total UI bill, which, in turn, equals z times the number of unmatched workers in the economy.[1] Under System B, taxes are personalized, and the typical firm's payment is proportional to the number of applications it receives (again, see footnote 1).

A third alternative that the authorities can adopt in order to guarantee an UI payment to the unemployed is System C. This specification does not involve taxes. Instead, the government requires firms to post a general schedule (w, z), where w denotes the wage paid to the employed worker, and z is a payment made (by the firm) to all workers who applied but were not offered a job. In this case, the typical firm j chooses (w^j, z^j), taking as given the strategies of its competitors, $(\tilde{w}, \tilde{z})$, in order to solve

$$\max_{w^j, z^j} \left\{ \left[1-(1-\theta)^n\right](y-w^j) - \left[n\theta - 1 + (1-\theta)^n\right] z^j \right\}$$

$$s.t.\ \frac{1-(1-\theta)^n}{n\theta} w^j + \left[1 - \frac{1-(1-\theta)^n}{n\theta}\right] z^j = \frac{1-\left(1-\tilde{\theta}\right)^n}{n\tilde{\theta}} \tilde{w} + \left[1 - \frac{1-\left(1-\tilde{\theta}\right)^n}{n\tilde{\theta}}\right] \tilde{z},$$

The firm's problem admits a similar interpretation as the ones under Systems A and B. The main difference is that now z is not a policy parameter, but a control variable of the firm (the same is true about $\tilde{z}$).

Having established the typical firm's problem under all possible systems, I now state and prove the main results of the Online Appendix in the following proposition. Subsequently, I discuss these results and the intuition behind them.

Proposition: Define $w^*(n, m), \pi^*(n, m)$ as the symmetric equilibrium wage and profits (respectively), in the finite directed search model with no taxes.

a) Under Systems A or B, the symmetric equilibrium wage is:

$$w^A(n,m) = \frac{n\left(1-\frac{1}{m}\right)^n y + m\left[1-\left(1+\frac{n}{m-1}\right)\left(1-\frac{1}{m}\right)^n\right] z}{m - \left(m+\frac{n}{m-1}\right)\left(1-\frac{1}{m}\right)^n}. \tag{1}$$

$$w^B(n,m) = \frac{n\left(1-\frac{1}{m}\right)^n y - \left[\left(m+\frac{n}{m-1}\right)\left(1-\frac{1}{m}\right)^n + \frac{n(m-1)}{m} - m\right] z}{m - \left(m+\frac{n}{m-1}\right)\left(1-\frac{1}{m}\right)^n}, \tag{2}$$

and, for any $n, m \geq 2$, we have $w^B(n,m) \leq w^*(n,m) < w^A(n,m)$.

[1] When workers apply to a firm with probability θ, the number of workers to whom that firm cannot offer a job is $\sum_{i=1}^{n} \binom{n}{i} (1-\theta)^{n-i} \theta^i (i-1)$, and one can verify that this expression is equal to $n\theta - 1 + (1-\theta)^n$. Since here there is one firm that receives workers' applications with probability θ (firm j), and $m-1$ firms that receive workers' applications with probability $\tilde{\theta}$ (the rest), the term in curly brackets, in the formula for τ^A, represents the economy-wide unemployment.

b) Under Systems A or B, the symmetric equilibrium profit is:

$$\pi^A(n,m) = \left[1-\left(1-\frac{1}{m}\right)^n\right](y-w^A) - \left[\frac{n}{m}-1+\left(1-\frac{1}{m}\right)^n\right]z, \tag{3}$$

$$\pi^B(n,m) = \left[1-\left(1-\frac{1}{m}\right)^n\right](y-w^B) - \left[\frac{n}{m}-1+\left(1-\frac{1}{m}\right)^n\right]z, \tag{4}$$

and, for any $n, m \geq 2$, we have $\pi^A(n,m) < \pi^B(n,m) < \pi^*(n,m)$.

c) Under System C, every pair $\left(w^C, z^C\right)$ that satisfies

$$w^C = \frac{n\left(1-\frac{1}{m}\right)^n y - \left[\left(m+\frac{n}{m-1}\right)\left(1-\frac{1}{m}\right)^n + \frac{n(m-1)}{m} - m\right]z^C}{m - \left(m+\frac{n}{m-1}\right)\left(1-\frac{1}{m}\right)^n}, \tag{5}$$

$$\pi^C = \left[1-\left(1-\frac{1}{m}\right)^n\right]\left(y-w^C\right) - \left[\frac{n}{m}-1+\left(1-\frac{1}{m}\right)^n\right]z^C \in [0, S(m,n)], \tag{6}$$

constitutes a symmetric equilibrium. The expression $S(n,m)$ denotes the expected surplus in the economy, which is fixed and given by

$$S(n,m) \equiv m\left[1-\left(1-\frac{1}{m}\right)^n\right]y.$$

Proof. The expressions $w^*(n,m), \pi^*(n,m)$ can be obtained simply by setting $z=0$ in either (1) or (2) (for w^*) and in either (3) or (4) (for π^*). The expression for the total expected surplus follows immediately if one observes that $1-(1-1/m)^n$ is the probability with which the typical firm receives at least one application, when workers apply to all firms with equal probability.

a) Consider first System A. The indifference constraint faced by firm j can be re-written as

$$[1-(1-\theta)^n]w^j = [1-(1-\theta)^n]z + \frac{\theta}{\tilde{\theta}}\left[1-\left(1-\tilde{\theta}\right)^n\right](\tilde{w}-z).$$

Replacing the term $[1-(1-\theta)^n]w^j$ from the equation above into firm j's profit, $\pi^j(w^j,\theta)$, allows one to write the firm's problem as a function only of the variable θ. More precisely:

$$\max_{\theta}\Big\{[1-(1-\theta)^n](y-z) - \frac{\theta}{\tilde{\theta}}\left[1-\left(1-\tilde{\theta}\right)^n\right](\tilde{w}-z)$$
$$-\frac{1}{m}\Big\{n\theta-1+(1-\theta)^n+(m-1)\left[n\tilde{\theta}-1+\left(1-\tilde{\theta}\right)^n\right]\Big\}z\Big\},$$

subject to $\tilde{\theta} = (1-\theta)/(m-1)$. Firm j's best response satisfies the first-order condition

$$n(1-\theta)^{n-1}(y-z) + \left[\frac{1-\left(1-\tilde{\theta}\right)^n}{\tilde{\theta}} - \theta \frac{n\tilde{\theta}\left(1-\tilde{\theta}\right)^{n-1} - 1 + \left(1-\tilde{\theta}\right)^n}{\tilde{\theta}^2(m-1)} \right] (z - \tilde{w}) + \frac{n}{m}\left[(1-\theta)^{n-1}\left(1-\tilde{\theta}\right)^{n-1}\right] z = 0$$

One can now impose the following symmetry conditions. First, all firms post the same wage, $w^j = \tilde{w} = w^A$. This, in turn, implies that all workers will apply to each firm with the same probability, $\theta = \tilde{\theta} = 1/m$. Once these observations are incorporated into the first-order condition, the latter becomes an equation with only one unknown, w^A. Solving with respect to this variable yields the formula reported in (1). The derivation of w^B in (2) follows identical steps.

Equation (1) reveals that $w^A > w^*$ will be satisfied *iff*

$$G(n,m) \equiv \left(1-\frac{1}{m}\right)^n \left(1+\frac{n}{m-1}\right) < 1. \tag{7}$$

Notice that, for any given $n \geq 2$, the function G is strictly increasing in m, since

$$\frac{\partial G(n,m)}{\partial m} = \frac{n(n-1)}{m^2(m-1)}\left(1-\frac{1}{m}\right)^{n-1} > 0.$$

Moreover, for any given $n \geq 2$, $\lim_{m\to\infty} G(n,m) = 1$. Since for any given $n \geq 2$, the function G is strictly increasing in m and bounded form above by the unit, the requirement in (7) holds true, and hence $w^A > w^*$ is established.

The proof of the fact that $w^* \geq w^B$ follows similar steps. However, this inequality is weak, because for $n = m = 2$ the multiplier of z in the numerator of the right-hand side in (2) equals zero. For any other value of n, m that term is positive, thus $w^* > w^B$.

b) The formulas reported in (3) and (4) follow immediately from part (a) and the definition of symmetric equilibrium profit. To show that $\pi^B < \pi^*$, notice that after some algebra, one can write

$$\pi^B(n,m) = \pi^* - \frac{1-\left(1+\frac{n}{m-1}\right)\left(1-\frac{1}{m}\right)^n}{m-\left(m+\frac{n}{m-1}\right)\left(1-\frac{1}{m}\right)^n}\, z. \tag{8}$$

The multiplier of z in (8), will be positive if and only if

$$1 > \left(1+\frac{n}{m-1}\right)\left(1-\frac{1}{m}\right)^n.$$

But this is precisely the condition that guarantees $w^A > w^*$ in the proof of part (a) (condition

(7)). Hence, one can conclude that $\pi^* > \pi^B$.

The last claim in part (b) is that $\pi^B > \pi^A$. In the symmetric equilibrium, the firms will always pay the same taxes, $[n/m - 1 + (1 - 1/m)^n]z > 0$. Therefore, the equilibrium profit will be higher under the system that leads to a lower equilibrium wage and, from part (a), this is Taxation System B.

c) Under System C, the indifference constraint faced by firm j can be re-written as

$$[1-(1-\theta)^n](w^j - z^j) + n\theta z^j = \frac{\theta}{\tilde{\theta}}\left[1-\left(1-\tilde{\theta}\right)^n\right](\tilde{w}-\tilde{z}) + n\theta\tilde{z}.$$

Replacing the left-hand side of the last expression into firm j's profit, allows one to re-write the firm's problem as

$$\max_{\theta}\left\{[1-(1-\theta)^n]y - n\theta\tilde{z} - \frac{\theta}{\tilde{\theta}}\left[1-\left(1-\tilde{\theta}\right)^n\right](\tilde{w}-\tilde{z})\right\},$$

subject to $\tilde{\theta} = (1-\theta)/(m-1)$. Firm j's best response satisfies the first-order condition

$$n(1-\theta)^{n-1}y + \left[\frac{1-\left(1-\tilde{\theta}\right)^n}{\tilde{\theta}} - \theta\frac{n\tilde{\theta}\left(1-\tilde{\theta}\right)^{n-1} - 1 + \left(1-\tilde{\theta}\right)^n}{\tilde{\theta}^2(m-1)}\right](\tilde{z}-\tilde{w}) - n\tilde{z} = 0.$$

One can now impose symmetry: $w^j = \tilde{w} = w^C$, $z^j = \tilde{z} = z^C$, and $\theta = \tilde{\theta} = 1/m$ into the first-order condition. After some algebra the result reported in (5) emerges.

Clearly, we have only one equation in order to characterize two equilibrium objects, $\left(w^C, z^C\right)$. Every pair $\left(w^C, z^C\right)$ that satisfies (5) is part of an equilibrium, as long as it leads to positive expected profit for the firms and positive expected utility for the workers. This is guaranteed by the condition in (6). □

The results described in parts (a) and (b) of the proposition admit similar interpretations as their counterparts in Proposition 1 of the main text. As I discussed there, a higher z tends to put downward pressure on equilibrium wages, because it implies higher taxes for the typical firm, and it increases the workers' outside option, so that a firm can be attractive even if it does not post a very high wage. At the same time, introducing unemployment benefits induces firms to be more aggressive in their wage setting strategy: when z is high, a firm that offers a high wage is extremely attractive because, even though workers understand that this firm will get many applications, they also understand that being unemployed is not such a bad outcome (precisely because the unemployment benefit is high). Under System A, the latter force prevails because firms can increase wages and become very attractive at the expense of other firms who will equally share the UI bill. Hence, in equilibrium we obtain $w^A > w^*$. Under System B, firms realize that a high w will lead to very high personalized taxes, and this minimizes the magnitude of the third force, so that, in equilibrium, $w^B < w^*$.

An importnant observation that stems from part (b) of the proposition is that, even under the personalized tax System B, the firms do not fully internalize the externality that characterizes this environment, in the sense that π^B depends on z, and $\pi^B < \pi^*$, for all $z > 0$. Recall from Proposition 1 in the main text that this is not the case in the large market environment: There, we saw that firms fully internalize the externality leading to an equilibrium profit which is independent of z and equal to $\bar{\pi}^*$. To better understand this result notice that in a finite economy firms have market power, and because of this they offer wages that are below the workers' expected marginal product.[2] Hence, even in the absence of UI and taxes (of any type), the small market is characterized by an inefficiency, which is associated with firms' market power. Naturally, this market power disappears in a large economy. As a result, System B, which is personalized and provides the correct incentives to firms in order to abstain from aggressive wage posting, achieves the efficient allocation in a market where all firms are price takers.

Part (c) of the proposition in this Online Appendix reveals that, under System C, equilibrium is indeterminate. A continuum of equilibria exist that are not payoff equivalent. In fact, as condition (6) indicates, any equilibrium profit in $[0, S(m, n)]$ (or, alternatively, any sharing rule of the surplus) can be supported. The indeterminacy of equilibrium that characterizes System C (in a finite market) is in the spirit of Coles and Eeckhout (2003). In that paper, sellers who possess one unit of an indivisible good, can charge a price that depends on ex-post realized demand (i.e. how many buyers show up at their store). What drives the indeterminacy result is that sellers can post advertisements that change the utility of buyers differently under different realizations of ex-post realized demand.[3] Since w^C, π^C are not uniquely pinned down, it is not possible to compare them with the equilibrium values w^i, π^i, $i \in \{A, B\}$. Nevertheless, as I show in the main text, this indeterminacy of equilibrium is a feature exclusively of the small market. As discussed in part (b) of Proposition 1 in the main text, in the large market case the equilibrium profit is uniquely pinned down.

References

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[2] For a more detailed discussion of this result, see Jacquet and Tan (2012).

[3] For a detailed discussion of this result, and the intuition behind it, see Coles and Eeckhout (2003) and Geromichalos (2014).