



Monetary policy and asset prices

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Abstract

The purpose of this paper is study the effect of monetary policy on asset prices. We study the properties of a monetary model in which a real asset is valued for its rate of return and for its liquidity. We show that money is essential if and only if real assets are scarce, in the precise sense that their supply is not sufficient to satisfy the demand for liquidity. Our model generates a clear connection between asset prices and monetary policy. When money grows at a higher rate, inflation is higher and the return on money decreases. In equilibrium, no arbitrage amounts to equating the real return of both objects. Therefore, the price of the asset increases in order to lower its real return. This negative relationship between inflation and asset returns is in the spirit of research in finance initiated in the early 1980s.

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1. Introduction

We know that monetary policy controls the money supply, which determines the rate of inflation, and hence the rate of return on (or the cost of holding) currency.¹ However, we also know that agents often manage portfolios with assets other than money in their daily transactions. Even

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¹ In simple models, in steady state the growth rate of the money supply pins down the inflation rate, and through the arbitrage condition known as the Fisher equation, this pins down the nominal interest rate; it does not matter if policy controls money, inflation or the interest rate, since any one determines the other two.

though these assets may differ in various properties, like liquidity or rate of return, and they may appear not even to be designed for transaction purposes, different assets in a portfolio maybe related in several ways. Lucas (1990) already points out possible interactions between liquidity and interest rates in an economy. Therefore, the intuitive concern arises: could monetary policy indeed affect the price or return of other assets in the economy? And if so, what would be the precise mechanism through which those effects would take place?

We use a model in the tradition of modern monetary theory, extended to include real assets in fixed supply just like the standard “trees” in Lucas (1978). However, these assets are not only stores of value in our model. They also compete with currency as a medium of exchange. We show that money is essential (i.e. monetary equilibria Pareto dominate non-monetary equilibria) if and only if real assets are scarce, in the precise sense that their supply is not sufficient to satisfy the demand for liquidity. In this case, real assets and money are concurrently used as means of payment, and an increase in inflation causes agents to want to move out of cash and into other assets. In equilibrium, this increases the price of these assets and lowers their rates of return.

Hence, the model predicts clearly that inflation reduces the return on other assets, which is something that has been discussed extensively in the finance literature for some time. Examples of papers that report this negative relationship are Fama (1981), Geske and Roll (2001), and Marshall (1992). Geske and Roll, for example, characterize the connection between asset returns and inflation as a puzzling empirical phenomenon that does not necessarily ascribe causality one way or the other. An early attempt to explain this finding in a general equilibrium framework was made by Danthine and Donaldson (1986), where money is assumed to yield direct utility. It does not seem right to analyze asset prices by putting assets in the utility function - would we take seriously as a “solution” to the equity premium puzzle a model where people “like” bonds more than stock? As opposed to this reduced-form monetary model we choose a setting in which the frictions that make money essential are explicitly described. Building on Lagos and Wright (2005), we provide a model based on micro foundations within which the effects of monetary policy on asset prices can be analyzed.

Several models based on Lagos and Wright (2005) have been created to study different questions related to the coexistence of multiple assets as media of exchange. For example, Lagos and Rocheteau (2006) allow capital to be traded in a decentralized market and focus on the issue of over-investment. They introduce real capital that can compete with money as a medium of exchange. Part of this capital may be productive but not liquid (in the sense that it cannot be used as a medium of exchange). Therefore, it will only be valued by its direct return derived from a storage technology. However, the other fraction can be used in decentralized trade and valued both for being productive and for its role as a medium of exchange. The object that we introduce is not related with returns due to productivity. Instead, our object is a real financial asset and all of it can be taken into the decentralized market. Thus, the main difference in our model lies in the asset-pricing implications. In their framework the price of the *liquid* capital has to be equal to that of the general good in the centralized market. In contrast, the price of our real financial asset will be determined endogenously and independently in equilibrium. This price reflects now both its return as a financial asset in terms of consumption and its role as a medium of exchange. They also conclude that when the return from the storage technology is higher than that as a medium of exchange agents tend to overaccumulate capital. This does not arise in our model.

Lagos (2005) builds an asset-pricing model in which financial assets (equity shares and one-period government risk-free real bills) are valued not only as claims to streams of consumption but also for their liquidity. In his model the price of an asset will be higher when is held for its exchange value, and its rate of return will be lower than it would if the asset was not used as

a medium of exchange. However, that framework is explicitly designed to address the risk-free rate and equity premium puzzles identified in Mehra and Prescott (1985), and more importantly, does not offer implications for monetary policy.

Like these papers, we allow alternative assets to compete as media of exchange, but our focus is on the competition between fiat money and real financial assets, and the implications of monetary policy on the price of and the rate of return on these assets. Assets are valued for what they yield, which includes direct rate of returns, as is standard in finance, and potentially some liquidity services, as is standard in monetary theory.² This paper provides a tractable model where money and other assets coexist, and where monetary policy affects equilibrium prices and rates of return on these assets in a straightforward and empirically relevant way.³

The rest of the paper is structured as follows. In Section 2 we describe the baseline model. In Section 3 we consider an economy without money and study the equilibrium properties of a model where the financial asset serves as the only medium of exchange. Section 4 introduces money and allows us to study the link between monetary policy and asset prices. Section 5 concludes.

2. The model

Time is discrete and each period consists of two subperiods. During the first subperiod trade occurs in a decentralized market, whereas in the second subperiod economic activity takes place in a centralized (Walrasian) market. There exists a $[0, 1]$ continuum of agents who live forever and discount future at rate $\beta \in (0, 1)$. Agents consume and supply labor in both subperiods. Preferences are given by $\mathcal{U}(x, h, X, H)$ where x and h (X and H) are consumption and labor in the decentralized (centralized) market, respectively. Following Lagos and Wright (2005), we use the specific functional form

$$\mathcal{U}(x, h, X, H) = u(x) - c(h) + U(X) - AH.$$

We assume that u and U are twice continuously differentiable with $u(0) = 0$, $u' > 0$, $u'(0) = \infty$, $U' > 0$, $u'' < 0$, and $U'' \leq 0$. To simplify the presentation we assume $c(h) = h$, but this is not important for our substantive results. Let q^* denote the efficient quantity, i.e. $u'(q^*) = 1$, and suppose there exists $X^* \in (0, \infty)$ such that $U'(X^*) = 1$ with $U(X^*) > X^*$.

During the first subperiod, economic activity is similar to the standard search model. Agents interact in a decentralized market and anonymous bilateral trade takes place. The probability of meeting someone is α . The first subperiod good, x , comes in many specialized varieties. Agents can transform labor into a special good on a one-to-one basis, and they do not consume the variety that they produce. When two agents, say i and j , meet in the decentralized market, there are three possibilities. The probability that i consumes what j produces but not vice versa (*single coincidence meeting*) is $\sigma \in (0, \frac{1}{2})$. By symmetry, the probability that j consumes what i produces but not vice versa is also σ . Finally, the probability that neither of the agents likes

² Applications of earlier monetary theory to finance include Bansal and Coleman (1996) and Kiyotaki and Moore (2005). In the former, the authors examine the joint distribution of asset returns, velocity, inflation, and money growth. In the latter, the analysis focusses on the interaction between liquidity, asset prices, and aggregate economic activity.

³ Recent work by Lester et al. (2007) extends our framework by modeling different acceptabilities for the underlying assets. Since their model builds on ours, the main results in our paper are also verified in Lester et al. However, their main contribution is to go a step further and obtain acceptability endogenously. Therefore, in equilibrium, assets have different liquidity properties.

what the other agent produces is $1 - 2\sigma$. In a single coincidence meeting, if i consumes what j produces, we will call i the buyer and j the seller.

During the second subperiod agents trade in a centralized market. With centralized trade it is irrelevant whether the good comes in one or many varieties. We assume that at the second subperiod all agents consume and produce the same general good. Agents in the centralized market can transform one unit of labor into w units of the general good, where w is a technological constant. We normalize that constant to $w = 1$ without loss of generality. It is also assumed that both the general and the special goods are non-storable.

There is another object, called (fiat) *money*, that is perfectly divisible and can be stored at any quantity $m \geq 0$. We let ϕ denote its value in the centralized market. The key new feature of our model is the introduction of a real asset. One can think of this asset as a *Lucas tree*. We assume that this tree lives forever, and that agents can buy its shares in the centralized market at price ψ . Let $T > 0$ denote the total stock of the real asset. We assume that T is exogenously given and constant. As opposed to fiat money, the asset yields a real return in terms of the general consumption good. We let the gross return (dividend) of the asset be denoted by $R > 0$. An agent that owns shares of the tree can consume the fruit (which here is just the general good) and sell the shares in the Walrasian market. She can also carry shares of the tree into the decentralized market in order to trade. Hence, in this economy money and the asset can potentially compete as a media of exchange.

3. Equilibrium in a model without money

We first consider an economy without money. Analyzing this simpler version of the model, provides intuition for the next section, where money and the asset can coexist as media of exchange. We begin with the description of the value functions, treating the prices and the distribution of asset holdings as given. These objects will be endogenously determined in equilibrium. Agents are allowed to keep any positive quantity of assets at home before they visit the decentralized market.⁴ The state variables for a representative agent are the units of the asset that she brings into the decentralized market, b , and the units of the asset that she keeps at home, a . To reduce notation we define $\omega \equiv (a, b)$. First, consider the value function in the centralized market for an agent with portfolio ω . We normalize the price of the general good to 1. The Bellman's equation is given by

$$W(\omega) = \max_{X, H, \omega_{+1}} \{U(X) - H + \beta V(\omega_{+1})\}$$

$$\text{s.t. } X + \psi(b_{+1} + a_{+1}) = H + (R + \psi)(b + a),$$

where b_{+1} , a_{+1} are the units of the asset carried into the next period's decentralized market and left at home, respectively.⁵ It can be easily verified that, at the optimum, $X = X^*$. Using this fact and replacing H from the budget constraint into W yields

$$W(\omega) = \max_{\omega_{+1}} \{U(X^*) - X^* - \psi(b_{+1} + a_{+1}) + (R + \psi)(b + a) + \beta V(\omega_{+1})\}. \quad (1)$$

⁴ As opposed to money, the asset here serves as a store of value. Moreover, the terms of trade for an agent may depend on the amount of the asset that she carries into the decentralized market. Therefore, it may be prudent to keep some assets at home. As we shall see in what follows, this is precisely what happens in equilibrium, whenever the supply of assets is sufficiently large to satisfy the demand for liquidity in the economy.

⁵ We impose non-negativity on all the control variables except H , which is allowed to be negative. Once equilibrium has been found, one can introduce conditions that guarantee $H \geq 0$ (see Lagos and Wright, 2005).

This expression implies the following results. First, the choice of ω_{+1} does not depend on ω . In other words, there are no wealth effects. This result relies on the quasi-linearity of \mathcal{U} . Second, W is linear,

$$W(\omega) = \Lambda + (R + \psi)(b + a).$$

We now turn to the terms of trade in the decentralized market. Consider a single-coincidence meeting between a buyer and a seller with state variables ω and $\tilde{\omega}$, respectively. We use the generalized Nash solution, where the bargaining power of the buyer is denoted by $\theta \in (0, 1]$. Let q represent the quantity of the special good and d_b the units of the asset that change hands during trade. The bargaining problem is

$$\max_{q, d_b} [u(q) + W(a, b - d_b) - W(\omega)]^\theta [-q + W(\tilde{a}, \tilde{b} + d_b) - W(\tilde{\omega})]^{1-\theta}$$

subject to $d_b \leq b$. Using the linearity property of W , the problem simplifies to

$$\max_{q, d_b} [u(q) - (R + \psi)d_b]^\theta [-q + (R + \psi)d_b]^{1-\theta}$$

subject to $d_b \leq b$. Clearly, the linearity of W implies that the solution to the bargaining problem does not depend on the variables a, \tilde{a}, \tilde{b} . It depends on b only if the constraint binds. The solution to this problem is a variation of the standard bargaining result obtained in this type of models.

Lemma 1. *The bargaining solution is the following:*

$$\begin{aligned} \text{If } b < b^*, \quad \text{then } \begin{cases} q(b) = \hat{q}(b), \\ d_b(b) = b. \end{cases} \\ \text{If } b \geq b^*, \quad \text{then } \begin{cases} q(b) = q^*, \\ d_b(b) = b^*, \end{cases} \end{aligned} \tag{2}$$

where b^* solves $(R + \psi)b = z(q^*) = \theta q^* + (1 - \theta)u(q^*)$, $\hat{q}(b)$ solves $(R + \psi)b = z(q)$, and $z(q)$ is defined by

$$z(q) \equiv \frac{\theta u'(q)q + (1 - \theta)u(q)}{\theta u'(q) + (1 - \theta)}. \tag{3}$$

Proof. It is straightforward to verify that the suggested solution satisfies the necessary first-order conditions, which are also sufficient in this problem. \square

Lemma 2. *For all $b < b^*$, we have $\hat{q}'(b) > 0$ and $\hat{q}(b) < q^*$.*

Proof. See Appendix A. \square

Next, consider the value function of an agent in the decentralized market. Let $F(b)$ be the distribution of asset holdings in this market. We set $\xi \equiv \alpha\sigma$. The Bellman's equation is⁶

⁶ The first term is the payoff from buying $q(b)$ and going to the centralized market with state variables $(a, b - d_b(b))$. The second term is the expected payoff from selling $q(\tilde{b})$ and going to the centralized market with state variables $(a, b + d_b(\tilde{b}))$. Both expressions reflect the fact that the only relevant variable for the determination of the terms of trade is the amount of assets that the buyer brings into the decentralized market. The last term is the payoff from going to the centralized market without trading in the decentralized market.

$$V(\omega) = \xi \{u[q(b)] + W[a, b - d_b(b)]\} + \xi \int \{-q(\tilde{b}) + W[a, b + d_b(\tilde{b})]\} dF(\tilde{b}) + (1 - 2\xi)W(\omega).$$

Using (1) and the linearity property of W , we can rewrite the decentralized market value function as

$$V(\omega) = \kappa + v(\omega) + \max_{\omega_{+1}} \{[-\psi + \beta(R + \psi_{+1})](b_{+1} + a_{+1}) + \beta\xi \{u[q_{+1}(b_{+1})] - (R + \psi_{+1})d_{b,+1}(b_{+1})\}\}, \tag{4}$$

where $\kappa = U(X^*) - X^* + \xi \int \{-q(\tilde{b}) + (R + \psi)d_b(\tilde{b})\} dF(\tilde{b})$ is a constant and

$$v(\omega) = \xi \{u[q(b)] - (R + \psi)d_b(b)\} + (R + \psi)(b + a).$$

In order to study the optimal behavior of the agent we focus on the term inside the maximum operator in (4). We define this term as $J(\omega_{+1})$ and refer to it as the objective function. This function consists of two terms. The term $-\psi + \beta(R + \psi_{+1})$ is the net gain of carrying an additional unit of the asset from the centralized market of the current period into the centralized market of the next period. Sometimes we refer to the negative of this term as the cost of carrying the asset across periods. The expression $u[q_{+1}(b_{+1})] - (R + \psi_{+1})d_{b,+1}(b_{+1})$ is the gain from trade, of carrying an additional unit of the asset into the next period’s decentralized market. We refer to this term as the surplus of the buyer. Next, we show that in any equilibrium the net gain of carrying the asset across periods is non-positive.

Lemma 3. *In any equilibrium, $\psi \geq \beta(R + \psi_{+1})$.*

Proof. See Appendix A. \square

The following lemma establishes some important properties of the optimal choice of the agent. To ease the presentation, we assume that $e(q) \equiv \xi u'(q)/z'(q) + 1 - \xi$ is a strictly decreasing function of q in the range $(0, q^*)$. This assumption insures that there exists a unique choice of b_{+1} that maximizes the objective function.⁷ As in Lagos and Wright (2005), conditions that guarantee the monotonicity of $e(q)$ are $\theta \approx 1$ or u' is log-concave.

Lemma 4. *Assume that $e'(q) < 0$ for all $q \in (0, q^*)$. In every period, the optimal choice of b_{+1} is unique and satisfies $b_{+1} \in (0, \bar{b}]$, where $\bar{b} \equiv \{b : (R + \psi)b = z(\bar{q})\}$, and \bar{q} solves $u'(\bar{q}) = z'(\bar{q})$. The optimal choice of a_{+1} satisfies*

$$a_{+1} = \begin{cases} 0, & \text{if } \psi > \beta(R + \psi_{+1}), \\ \in \mathfrak{R}_+, & \text{if } \psi = \beta(R + \psi_{+1}). \end{cases} \tag{5}$$

Proof. See Appendix A. \square

According to Lemma 4, the agent’s optimal choice of a_{+1} depends only on whether the cost of transferring assets across time is zero or positive. The optimal choice of b_{+1} never exceeds \bar{b} ,

⁷ If the set of maximizers of $J(\omega_{+1})$ with respect to b_{+1} is not a singleton, one can still draw conclusions about the model at the cost of a higher level of complexity. To avoid that, we assume that $e'(q) < 0$ for all $q \in (0, q^*)$. Notice that this assumption is sufficient but not necessary for the existence of a unique optimal choice of b_{+1} .

and it is equal to \bar{b} only if $\psi - \beta(R + \psi_{+1}) = 0$.⁸ In this case, the quantity of the special good purchased in the decentralized market is \bar{q} . Sometimes, we refer to this value of q as the constrained efficient quantity. For every $\theta < 1$, we have $\bar{q} < q^*$, and so (unconstrained) efficiency is never obtained in equilibrium.⁹ Another implication of Lemma 4 is that the first order condition with respect to b_{+1} is always satisfied with equality,

$$-\psi + \beta(R + \psi_{+1})e[\hat{q}_{+1}(b_{+1})] = 0. \quad (6)$$

Finally, uniqueness of the optimal choice of b_{+1} implies that the distribution of asset holdings is degenerate (i.e. all the agents carry the same amount of the asset into the decentralized market).

Definition 1. An equilibrium for this economy is a value function $V(\omega)$ that satisfies Bellman's equation, a solution to the bargaining problem given by $d_b(b) = b$ and $q(b) = \hat{q}(b)$, and a bounded path of ψ such that (5) and (6) hold at every date with $a + b = T$.

The key factor which determines whether a_{+1} is equal to or greater than zero in equilibrium is the total stock of the asset. High T (in a sense to be clarified below) leads to equilibria with $a_{+1} > 0$, while low T induces agents to carry the whole amount of assets they own into the decentralized market.

Consider first equilibria with $a_{+1} = 0$. In this case, we can replace b_{+1} with T in both (6) and the solution to the bargaining problem. The latter can be rewritten as

$$(R + \psi)T = z(q). \quad (7)$$

Since (7) holds at every date, we can solve for ψ , ψ_{+1} and insert these expressions into (6). This yields

$$z(q) = \beta z(q_{+1})e(q_{+1}) + RT, \quad (8)$$

where it is understood that $q = q(T)$ and $q_{+1} = q_{+1}(T)$. Equation (8) is a first-order difference equation in q . Equilibrium could be redefined as a path of q that stays in $(0, \bar{q}]$ and satisfies (8). We focus on steady state equilibria, which are defined to be solutions to (8) with the additional requirement that $q_{+1} = q$.¹⁰

We analyze the steady state of this version of the model without money in Appendix A. We show that there exists a level of supply of the financial assets, \bar{T} , such that, for every $T > \bar{T}$, the agent brings into the match just enough assets to purchase \bar{q} and leaves the rest at home. The analysis also reveals that in any equilibrium the price of the asset satisfies $\psi \geq \bar{\psi}$, where $\bar{\psi}$ is the only value that guarantees a bounded path for the equilibrium price of the asset. It is straightforward to verify that $\bar{\psi}$ is the present value of the stream of dividends of the asset or, alternatively, the fundamental value of the asset. When $T < \bar{T}$, we have $\psi > \bar{\psi}$, i.e. the price of the asset exceeds its fundamental value. The intuition behind this result is that when T is relatively low the price of the asset reflects its fundamental value as well as the asset's value as a medium of exchange. On the other hand, when $T \geq \bar{T}$ the agents choose $a_{+1} > 0$ and so the

⁸ It is straightforward to verify that \bar{b} maximizes the surplus of the buyer.

⁹ This result is in the spirit of the Hosios' condition for efficiency (see Hosios, 1990). According to this, the bargaining solution is efficient if it splits the surplus so that each participant is compensated for her contribution to the match. In this model the whole surplus is due to the buyer. This implies that, if $\theta < 1$, the buyer does not get all the surplus from the match and consequently never carries the socially optimal quantity b^* into the decentralized market.

¹⁰ A detailed discussion on dynamics in this class of models is presented in Lagos and Wright (2003).

value of the asset as a medium of exchange has to be zero (if it was positive it would never be optimal to keep any assets at home). We refer to the difference between ψ and $\bar{\psi}$ as the liquidity premium of the asset. The next proposition summarizes the most important results of this section.

Proposition 1. *Consider the model without money and assume that $e'(q) < 0$ for all $q \in (0, q^*)$. For every $T > 0$ there exists a unique steady state equilibrium where the real asset serves as a medium of exchange. If $T \geq \bar{T}$, constrained efficiency is achieved, i.e. $q^S = \bar{q}$. For this range of T we have $\psi = \bar{\psi}$ and $a_{+1} = T - \bar{T}$. If $T < \bar{T}$, then $a_{+1} = 0$, $q^S \in (0, \bar{q})$, and the asset bears a positive liquidity premium equal to $\psi - \bar{\psi}$. Moreover, q^S is strictly increasing in T and ψ is strictly decreasing.*

4. Money and real assets

In this section we introduce a new object, called (fiat) money, which is perfectly divisible and can be stored at any positive quantity $m \geq 0$. Agents can bring any amount of this object into the decentralized market in order to trade. We assume that the supply of money evolves according to $M_{+1} = (1 + \mu)M$, where μ is a constant. The new money is injected into the economy as a lump-sum transfer after the agents leave the centralized market. The state variables for the agent are m, a and b .¹¹ To reduce notation we redefine $\omega \equiv (m, a, b)$. First, consider the value function in the centralized market. The Bellman's equation is

$$W(\omega) = \max_{X, H, \omega_{+1}} \{U(X) - H + \beta V(\omega_{+1})\}$$

$$\text{s.t. } X + \phi m_{+1} + \psi(b_{+1} + a_{+1}) = H + \phi m + (R + \psi)(b + a),$$

where, except from the new variables m, m_{+1} and the value of money ϕ , everything is the same as in the previous section. Replacing H from the budget constraint and using the fact that at the optimum $X = X^*$, we have

$$W(\omega) = \max_{\omega_{+1}} \{U(X^*) - X^* - \phi m_{+1} - \psi(b_{+1} + a_{+1}) + \phi m + (R + \psi)(b + a) + \beta V(\omega_{+1})\}. \quad (9)$$

As in Section 3, the quasi-linearity of \mathcal{U} rules out wealth effects, and W is linear,

$$W(\omega) = \Lambda + \phi m + (R + \psi)(b + a).$$

We now turn to the terms of trade in the decentralized market. Consider a single-coincidence meeting between a buyer and a seller with state variables ω and $\tilde{\omega}$, respectively. Let d_m represent the amount of money spent in order to buy the special good. The generalized Nash bargaining problem is given by

$$\max_{q, d_m, d_b} [u(q) + W(m - d_m, a, b - d_b) - W(\omega)]^\theta$$

$$\times [-q + W(\tilde{m} + d_m, \tilde{a}, \tilde{b} + d_b) - W(\tilde{\omega})]^{1-\theta}$$

¹¹ Notice that while a stands for the amount of the real asset that the agent keeps at home, there is no analogous variable for money. Unlike the asset, which serves as a store of value, fiat money is only valued for its liquidity services. Therefore, the agent never has an incentive to leave a positive amount of money at home before visiting the decentralized market.

s.t. $d_m \leq m$ and $d_b \leq b$. Using the linearity property of W , this problem simplifies to

$$\max_{q, d_m, d_b} [u(q) - \phi d_m - (R + \psi)d_b]^\theta [-q + \phi d_m + (R + \psi)d_b]^{1-\theta}$$

s.t. $d_m \leq m$ and $d_b \leq b$. Clearly, the solution to the bargaining problem can only depend on the money and asset holdings of the buyer.

Lemma 5. Define total real balances as $\pi \equiv \phi m + (R + \psi)b$. The bargaining solution is the following:

$$\begin{aligned} \text{If } \pi < \pi^*, \quad \text{then } & \begin{cases} q(m, b) = \hat{q}(\pi), \\ d_m(m, b) = m, \quad d_b(m, b) = b. \end{cases} \\ \text{If } \pi \geq \pi^*, \quad \text{then } & \begin{cases} q(m, b) = q^*, \\ \phi d_m(m, b) + (R + \psi)d_b(m, b) = z(q^*), \end{cases} \end{aligned} \tag{10}$$

where $z(q)$ is defined by (3), π^* is the set of pairs (m, b) such that $\phi m + (R + \psi)b = z(q^*)$, and $\hat{q}(\pi)$ solves $\pi = z(q)$.

Proof. It can be easily verified that the suggested solution satisfies the first-order conditions, which are also sufficient in this problem. \square

Lemma 5 reveals that all pairs (m, b) that yield the same π buy equal quantity in the match. In other words, the seller is only interested in the term $\phi m + (R + \psi)b$ that the buyer carries and not in the composition of the portfolio.

Lemma 6. For all $\pi < \pi^*$, we have $\hat{q}'(\pi) > 0$ and $\hat{q}(\pi) < q^*$.

Proof. The proof is identical to the one of Lemma 2. \square

Next, consider the value function in the decentralized market. We redefine $F(m, b)$ to be the joint distribution of money and asset holdings in this market. The Bellman’s equation is¹²

$$\begin{aligned} V(\omega) = & \xi \{ u[q(m, b)] + W[m - d_m(m, b), a, b - d_b(m, b)] \} \\ & + \xi \int \{ -q(\tilde{m}, \tilde{b}) + W[m + d_m(\tilde{m}, \tilde{b}), a, b + d_b(\tilde{m}, \tilde{b})] \} dF(\tilde{m}, \tilde{b}) \\ & + (1 - 2\xi)W(\omega). \end{aligned}$$

Using (9) and the linearity property of W , we can rewrite the value function in the decentralized market as

$$\begin{aligned} V(\omega) = & \lambda + v(\omega) + \max_{\omega_{+1}} \{ (-\phi + \beta\phi_{+1})m_{+1} + [-\psi + \beta(R + \psi_{+1})](b_{+1} + a_{+1}) \\ & + \beta\xi \{ u(q_{+1}) - \phi_{+1}d_{m,+1} - (R + \psi_{+1})d_{b,+1} \} \}, \end{aligned} \tag{11}$$

¹² The first term is the payoff from buying $q(m, b)$ and going to the centralized market with state variables $(m - d_m(m, b), a, b - d_b(m, b))$. The second term is the expected payoff from selling $q(\tilde{m}, \tilde{b})$ and going to the centralized market with state variables $(m + d_m(\tilde{m}, \tilde{b}), a, b + d_b(\tilde{m}, \tilde{b}))$. Both expressions reflect the fact that the only relevant variables for the determination of the terms of trade are the money and asset holdings of the buyer in the decentralized market. The last term is the payoff from going to the centralized market without trading in the decentralized market.

where

$$\lambda = U(X^*) - X^* + \beta\phi_{+1}\mu M + \xi \int \{-q(\tilde{m}, \tilde{b}) + \phi d_m(\tilde{m}, \tilde{b}) + (R + \psi)d_b(\tilde{m}, \tilde{b})\} dF(\tilde{m}, \tilde{b})$$

is a constant,

$$v(\omega) = \xi \{u[q(m, b)] - \phi d_m(m, b) - (R + \psi)d_b(m, b)\} + \phi m + (R + \psi)(b + a),$$

and it is understood that q_{+1} , $d_{b,+1}$, and $d_{m,+1}$ are all functions of $m_{+1} + \mu M$ and b .¹³

As in the previous section, we define the term inside the maximum operator in (11) as $J(\omega_{+1})$, and refer to it as the objective function. This function consists of three terms. The first term is the cost of carrying m_{+1} units of money from period to period, and the second term is the cost of carrying the real asset across periods. The last term is the discounted, expected surplus of the buyer.¹⁴

Lemma 7. *In any equilibrium $\phi \geq \beta\phi_{+1}$, and $\psi \geq \beta(R + \psi_{+1})$.*

Proof. The proof is identical to the one of Lemma 3. \square

We now study the optimal behavior of the agent. The choice variable a_{+1} enters the objective function linearly, and it is multiplied by the cost of holding the asset. Therefore, the optimal choice of a_{+1} is still given by (5). The next lemma characterizes the optimal choice of m_{+1} and b_{+1} . To keep the analysis simple, we assume that the function $e(q) \equiv \xi u'(q)/z'(q) + 1 - \xi$ is strictly decreasing for all $q \in (0, q^*)$.

Lemma 8. *Assume that $e'(q) < 0$ for all $q \in (0, q^*)$. The optimal choices of m_{+1} and b_{+1} satisfy $\phi(m_{+1} + \mu M) + (R + \psi)b_{+1} = \pi_{+1} \in (0, \bar{\pi}]$. The optimal choice is unique in terms of π_{+1} .*

Proof. See Appendix A. \square

Lemma 8 indicates that all agents enter the decentralized market with the same total real balances π_{+1} . In general, one cannot determine the choices of m_{+1} and b_{+1} separately. However, the bargaining solution implies that $z(\hat{q}) = \pi$. Therefore, the fact that m_{+1} and b_{+1} are not uniquely determined is not a problem, since the real balances are sufficient to determine q in equilibrium. Similarly, the joint distribution function $F(m, b)$ is not degenerate in terms of (m, b) but is degenerate if we express it in terms of real balances, i.e. $\tilde{F}(\pi)$. Finally, the first-order conditions with respect to m_{+1} and b_{+1} are given by

$$-\phi + \beta\phi_{+1}e(\hat{q}_{+1}) \leq 0, = 0 \quad \text{if } m_{+1} > 0, \tag{12}$$

$$-\psi + \beta(R + \psi_{+1})e(\hat{q}_{+1}) = 0. \tag{13}$$

We now provide a definition of equilibrium for the economy described in this section.¹⁵

¹³ Under the assumption of changes in money supply, the money holdings in the next period of an agent that chooses m_{+1} in the current period's centralized market are $m_{+1} + \mu M$.

¹⁴ Notice that the surplus of the buyer can be expressed in terms of π_{+1} only.

¹⁵ As opposed to the supply of the real asset, the supply of money can grow over time, and so a bounded path of ϕ is not enough to guarantee that the real money balances, ϕM , are also bounded. Instead of that, we require a path of ϕ such that the sequence $\{(\phi M)_t\}_{t=0}^{\infty}$ is bounded.

Definition 2. An equilibrium for the economy with money and the real asset is a value function $V(\omega)$ that satisfies Bellman’s equation, a solution to the bargaining problem given by $d_m(m, b) = m$, $d_b(m, b) = b$ and $q(m, b) = \hat{q}(\pi)$, a path of ϕ , and a bounded path of ψ such that (5), (12) and (13) hold at every date with $a + b = T$ and $m = M$, and the sequence $\{(\phi M)_t\}_{t=0}^\infty$ is bounded.

We now turn to the characterization of equilibria. The following lemma establishes a useful result that holds for equilibria with $a_{+1} = 0$.

Lemma 9. *In any steady state equilibrium with $a_{+1} = 0$, $\psi = \psi_{+1}$ and*

$$\frac{\phi}{\phi_{+1}} = 1 + \mu. \tag{14}$$

Proof. See Appendix A. \square

According to the stationary analysis in Appendix A, if $T \geq \bar{T}$, the agent carries just enough units of the asset to purchase \bar{q} and keeps the rest at home.¹⁶ One might suggest that the agent can keep some more assets at home (this is still optimal since $\psi = \beta(R + \psi_{+1})$) and carry some money into the decentralized market instead. This should be equivalent to bringing no money and $b_{+1} = \bar{T}$, as long as the new composition of the agent’s portfolio, say (\bar{m}, \bar{b}) , satisfies $\pi = \phi\bar{m} + (R + \psi)\bar{b} = z(\bar{q})$. The definition of the objective function reveals that this argument is not accurate. The pairs $(0, \bar{T})$ and (\bar{m}, \bar{b}) yield the same surplus in the match, but the cost terms associated with each pair might differ. In particular, since here $\psi = \beta(R + \psi_{+1})$, the cost of carrying the asset across periods is zero. Consequently, the agent would choose $m_{+1} > 0$ only if the cost of holding money is also zero, i.e. $\phi = \beta\phi_{+1}$. Using (14) this condition reduces to $\mu = \beta - 1$, i.e. the monetary authority should follow the Friedman Rule. We conclude that for economies with positive nominal interest rate and $T \geq \bar{T}$ there do not exist monetary equilibria.

If we allow for economies with zero nominal interest rate, then the Friedman Rule can be followed, and there exists a monetary equilibrium. However, in this case money does not add anything to welfare. Since money and the real asset are equally costly, agents substitute the asset with money in the decentralized market. Regardless of the composition of the agents’ portfolio, the equilibrium quantity always satisfies $q^S = \bar{q}$. Therefore, if $T \geq \bar{T}$, we say that there is no *essential* role for money.

Next, consider $T < \bar{T}$. In this case the real asset is not enough to satisfy the demand for liquidity in the economy, and so money has an *essential* role. Focussing on steady state equilibria and using Lemma 9, Eqs. (12) and (13) become

$$e(q^S) = \frac{1 + \mu}{\beta}, \tag{15}$$

$$\psi = \frac{\beta}{1 - \beta e(q^S)} Re(q^S). \tag{16}$$

Equation (15) implies that the steady state quantity is negatively related to the growth rate of money supply.¹⁷ Moreover, (15) and (16) together imply

$$\psi = \psi(\mu) = \frac{(1 + \mu)R}{-\mu}. \tag{17}$$

¹⁶ If $T = \bar{T}$, then $a_{+1} = 0$.

¹⁷ As we pointed out in the previous section, this result does not rely on the assumption that $e'(q) < 0$ for all $q \in (0, q^*)$.

Equation (17) provides an expression for the price of the real asset as a function of the policy rule. Since ψ cannot be negative we impose the restriction $\mu < 0$. The reason for this result is that in this model the return of the real asset is always positive, and its liquidity properties are equivalent to those of money. By no arbitrage, money and the financial asset must have the same rates of return. Therefore, money can only be valued if it yields a positive return, that is, if the monetary authority deflates the economy. If we were to support positive inflation, the liquidity properties of money and the financial asset should not be the same. This result can be found in Lester et al. where the authors provide money with a liquidity advantage. Thus, the rate of return on the real asset can be higher to make up for the liquidity premium disadvantage, and one can have positive inflation.¹⁸ For every $\mu \in (\beta - 1, 0)$, we have $\psi'(\mu) > 0$ and $\psi''(\mu) > 0$. As $\mu \rightarrow 0$ (constant money supply), the price of the real asset goes to infinity, and as $\mu \rightarrow \beta - 1$, $\psi \rightarrow \bar{\psi}$, which is the fundamental value of the real asset.¹⁹

The previous asset pricing equation can be interpreted as follows: As μ increases, inflation is higher and the rate of return on money, $\phi_{+1}/\phi - 1$, decreases. In equilibrium, no arbitrage implies that the rate of return on both objects has to be the same. Therefore, ψ increases in order to lower the rate of return on the asset, $(\psi + R)/\psi - 1 = R/\psi \equiv \rho_a$. It is straightforward to verify that as $\mu \rightarrow 0$, $\rho_a \rightarrow 0$ and as $\mu \rightarrow \beta - 1$, $\rho_a \rightarrow (1 - \beta)/\beta$.

It turns out that not all $\mu \in (\beta - 1, 0)$ are consistent with a monetary equilibrium. For any given $T < \bar{T}$, define ψ^T and $q^{S,T}$ as the (unique) steady state solution for the economy without money. We show in Appendix A that the range of monetary policies for which a monetary equilibrium can be supported is $(\beta - 1, \mu^T]$, where $\mu^T \equiv \{\mu : \frac{1+\mu}{-\mu}R = \psi^T\}$.²⁰ Figure 1 depicts the asset price as a function of μ for this region.

For every $\mu \in (\beta - 1, \mu^T]$, Eq. (15) implies that $q^S \geq q^{S,T}$, and so the introduction of money in the economy improves welfare. If, on the other hand, $\mu > \mu^T$, then in any monetary equilibrium the steady state quantity would be lower than the quantity associated with the non-monetary equilibrium. Agents realize that and choose not to carry any amount of money. We summarize these results in the following proposition.

Proposition 2. *Consider the model with money and the real asset. Assume that $e'(q) < 0$ for all $q \in (0, q^*)$. If $T \geq \bar{T}$, then $q^S = \bar{q}$, $\psi = \bar{\psi}$, and $a_{+1} = T - \bar{T}$. For this range of T , the equilibrium is always non-monetary, assuming that $\mu > \beta - 1$. If $T < \bar{T}$, money has an essential role and the range of policies consistent with a monetary equilibrium is $(\beta - 1, \mu^T]$, where μ^T is strictly decreasing in T . In every monetary equilibrium the policy rule determines ψ and q^S . The former is increasing in μ , and the latter is decreasing. The model predicts a negative re-*

¹⁸ Another possibility for supporting inflation in equilibrium is to model one-period trees instead of our infinitely lived ones. The rate of return on a tree that lives forever is given by $(\psi_{+1} + R)/\psi - 1 = (\psi + R)/\psi - 1 = R/\psi > 0$. Hence, a monetary equilibrium requires $\phi_{+1}/\phi - 1 > 0$, which is equivalent to $-\mu/(1 + \mu) > 0$. The last inequality holds if and only if $\mu < 0$. The assumption that lies behind this result is the infinite life of the tree. To see why this is true, consider a tree that lives only for one period. Under this specification, the return of the real asset is given by $R/\psi - 1$, which might be negative. The analogue of (17) is now $\psi = R(1 + \mu)$. Hence, positive values of μ can be consistent with equilibria, in which the return on both money and the real asset is negative.

¹⁹ Like in the previous section, when $T < \bar{T}$ (and $\mu > 1 - \beta$), the asset bears a positive liquidity premium. Naturally, money also bears a liquidity premium in every monetary equilibrium, since its fundamental value is zero.

²⁰ This restriction holds for all T , not only for $T < \bar{T}$. When $T \geq \bar{T}$, we have $\psi = \bar{\psi}$, and consistency requires $\mu^T = \beta - 1$. Hence, for this specific region of T , the range of admissible policies becomes the empty set. However, if we allow for cases with zero nominal interest rate, the lower bound of admissible policies includes $\beta - 1$ and the relevant set is a singleton, namely the Friedman Rule.

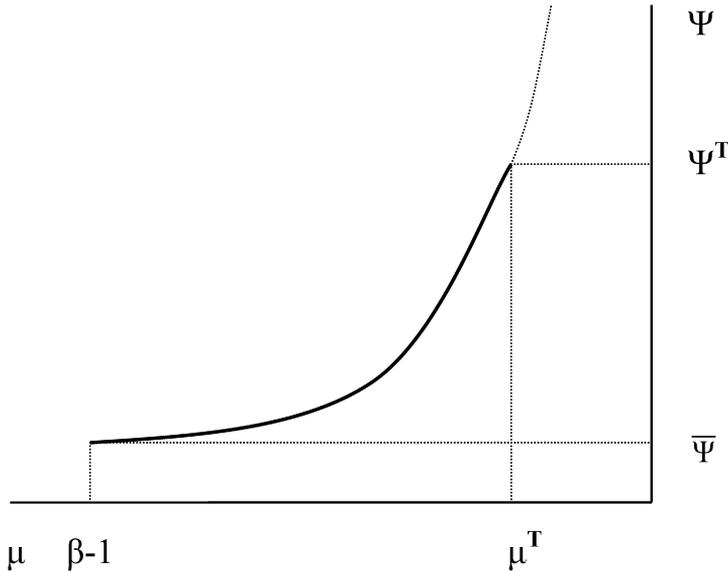


Fig. 1. The asset price as a function of μ .

relationship between inflation and asset returns. For every $\mu \in (\beta - 1, \mu^T)$, the introduction of money improves welfare. As $\mu \rightarrow \beta - 1$, $\psi \rightarrow \bar{\psi}$ and $q^S \rightarrow \bar{q}$. Therefore, whenever the supply of the asset is not sufficiently large to satisfy the demand for liquidity, constrained efficiency is achieved only if the Friedman Rule is followed.

5. Conclusions

In this paper we have studied the properties of a model in which money and a real financial asset compete as media of exchange. Focussing on equilibria with positive nominal interest rate, the key factor that determines whether money circulates or not is the aggregate supply of the asset. If the stock of the asset is sufficiently large to satisfy the liquidity needs of the economy, money has no value and the asset circulates as the only medium of exchange. If not, money and the asset are concurrently used as means of payment. Monetary equilibria Pareto dominate non-monetary ones, and so money has an essential role. Regarding monetary policy, welfare is negatively related to the growth rate of money (and therefore inflation). Constrained efficiency requires deflating the economy at the rate of time preference.

For equilibria where both money and the real asset circulate, the model delivers an explicit connection between the price of the asset and the policy rule. Specifically, the market price of the asset is a strictly increasing function of the growth rate of money. The interpretation of this result is the following: When money grows at a higher rate, inflation is higher and the return on money decreases. Since in our framework both assets have the similar liquidity properties, in equilibrium the rate of return on both objects has to be the same. Therefore, the price of the asset increases in order to lower its real return. This negative relationship between inflation and asset returns is consistent with empirical findings in finance literature initiated in the early 1980s by Fama, Geske, and Roll.

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Appendix A

Proof of Lemma 2. Using the definition of $\hat{q}(b)$ and applying the implicit function theorem in (3) yields

$$\hat{q}'(b) = \frac{(R + \psi)}{z'(\hat{q})} = \frac{(R + \psi)(\theta u' + 1 - \theta)^2}{u'(\theta u' + 1 - \theta) - \theta(1 - \theta)(u - q)u''}.$$

Hence, for all $b < b^*$, we have $\hat{q}'(b) > 0$. Moreover, it can be easily checked that

$$\lim_{b \rightarrow b^*} \hat{q}(b) = q^*.$$

We conclude that $\hat{q}(b) < q^*$ for all $b < b^*$. \square

Proof of Lemma 3. The budget constraint of the centralized market implies that the agent can increase the hours worked in period t by dH_t and get $db_{t+1} = \frac{dH_t}{\psi_t}$ units of the asset.²¹ In the next period the agent can decrease the amount of hours worked by $dH_{t+1} = -(R + \psi_{t+1})db_{t+1} = -(R + \psi_{t+1})\frac{dH_t}{\psi_t}$. The net utility gain of doing so is

$$dU_t = -dH_t + \beta dH_{t+1} = -dH_t \left[1 - \beta \left(\frac{R + \psi_{t+1}}{\psi_t} \right) \right].$$

Clearly, $\psi < \beta(R + \psi_{t+1})$ implies that $dU_t > 0$, and so in any equilibrium $\psi \geq \beta(R + \psi_{t+1})$. \square

Proof of Lemma 4. From (2) and (4) we obtain

$$J_b(\omega_{+1}) = \begin{cases} -\psi + \beta(R + \psi_{+1}), & \text{if } b_{+1} \geq b^*, \\ -\psi + \beta(R + \psi_{+1})e[\hat{q}_{+1}(b_{+1})], & \text{if } b_{+1} < b^*. \end{cases} \quad (\text{A.1})$$

Lemma 3 implies that $J_b(\omega_{+1}) \leq 0$ for every $b_{+1} \geq b^*$. It is also straightforward to verify that $\lim_{b_{+1} \rightarrow b^*-} J_b(\omega_{+1}) < 0$ and $\lim_{b_{+1} \rightarrow 0} J_b(\omega_{+1}) > 0$. Moreover, for all $b_{+1} < b^*$, we have

$$J_{bb}(\omega_{+1}) = \beta(R + \psi_{+1})e'(q)\hat{q}'(b) < 0,$$

where the inequality follows from Lemma 2. Combining these observations, we conclude that there exists a unique $b_{+1} \in (0, b^*)$ that maximizes the objective function.

²¹ The agent can either carry these assets into the decentralized market or keep them at home. For the purpose of this proof this choice is irrelevant, so we assume without loss of generality that she carries the asset into the decentralized market. Hence, we write db_{t+1} instead of da_{t+1} .

To show that the optimal choice also satisfies $b_{+1} \leq \bar{b}$, we use the first-order condition which holds with equality

$$\psi = \beta(R + \psi_{+1})e[\hat{q}_{+1}(b_{+1})]. \tag{A.2}$$

Lemma 3 and (A.2) imply that $e[\hat{q}_{+1}(b_{+1})] \geq 1$, or equivalently $\hat{q}_{+1}(b_{+1}) \leq \bar{q}$. Then, using Lemma 2 and the definition of \bar{q} we can conclude that $b_{+1} \leq \bar{b}$.

Regarding the optimal choice of a_{+1} , notice that this variable enters the objective function linearly and it is multiplied by the term $-\psi + \beta(R + \psi_{+1})$. Hence, (5) is an immediate consequence of Lemma 3. \square

Proof of Lemma 8. First, it is easy to verify that $\lim_{b_{+1} \rightarrow 0} J_b(\omega_{+1}) > 0$. Hence, $b_{+1} > 0$, and so $\pi_{+1} > 0$ regardless of the optimal choice of money holdings. If $m_{+1} = 0$, then the proof coincides with that of Lemma 4. In what follows we consider solutions with $m_{+1} > 0$.

Consider any pair (m_{+1}, b_{+1}) such that $\pi_{+1} = \phi(m_{+1} + \mu M) + (R + \psi)b_{+1} \geq \pi^*$. From (10) and (11) we obtain

$$J_m(\omega_{+1}) = -\phi + \beta\phi_{+1} \leq 0, \tag{A.3}$$

$$J_b(\omega_{+1}) = -\psi + \beta(R + \psi_{+1}) \leq 0, \tag{A.4}$$

where both inequalities follow from Lemma 7.

Next, consider any pair (m_{+1}, b_{+1}) such that $\pi_{+1} < \pi^*$. For this range we have

$$J_m(\omega_{+1}) = -\phi + \beta\phi_{+1}e(\hat{q}_{+1}),$$

$$J_b(\omega_{+1}) = -\psi + \beta(R + \psi_{+1})e(\hat{q}_{+1}).$$

Since $m_{+1}, b_{+1} > 0$, both first-order conditions hold with equality, implying that

$$\frac{\phi}{\psi} = \frac{\phi_{+1}}{R + \psi_{+1}}. \tag{A.5}$$

Using (A.5) we can rewrite the objective function as

$$J(\omega_{+1}) = [-\psi + \beta(R + \psi_{+1})]a_{+1} + \left(\beta - \frac{\phi}{\phi_{+1}}\right)\pi_{+1} + \beta\xi \{u[\hat{q}_{+1}(\pi_{+1})] - \pi_{+1}\}.$$

Differentiation with respect to π_{+1} yields

$$J_\pi(\omega_{+1}) = \left(\beta - \frac{\phi}{\phi_{+1}}\right) + \beta\xi \left\{ \frac{u'[\hat{q}_{+1}(\pi_{+1})]}{z'[\hat{q}_{+1}(\pi_{+1})]} - 1 \right\}. \tag{A.6}$$

Using the assumption that $e'(q) < 0$ together with Lemma 6, it follows that the function $\frac{u'[\hat{q}_{+1}(\pi_{+1})]}{z'[\hat{q}_{+1}(\pi_{+1})]}$ is strictly decreasing in π_{+1} . This observation has two important implications. First, it can be shown that $\lim_{\pi_{+1} \rightarrow \pi^*} J_\pi(\omega_{+1}) < 0$. Second, for all $\pi_{+1} \in (0, \pi^*)$, we have $J_{\pi\pi}(\omega_{+1}) < 0$. Combining these results with (A.3) and (A.4), we can conclude that the optimal choice of π_{+1} is unique and it satisfies $\pi_{+1} \in (0, \pi^*)$.

Finally, (A.6) indicates that at the optimum

$$\beta\xi \left\{ \frac{u'[\hat{q}_{+1}(\pi_{+1})]}{z'[\hat{q}_{+1}(\pi_{+1})]} - 1 \right\} = \frac{\phi}{\phi_{+1}} - \beta \geq 0,$$

where the inequality follows from Lemma 7, and it implies that $\hat{q}_{+1}(\pi_{+1}) \leq \bar{q}$. Then, using Lemma 6 and the definition of \bar{q} , it is straightforward to verify that the optimal choice of total real balances satisfies $\pi_{+1} \leq \bar{\pi}$. \square

Proof of Lemma 9. In any equilibrium with $a_{+1} = 0$, we have

$$\phi M + (R + \psi)T = z(q^S) = \phi_{+1}M_{+1} + (R + \psi_{+1})T,$$

which implies that

$$M[\phi - (1 + \mu)\phi_{+1}] + T(\psi - \psi_{+1}) = 0 \quad (\text{A.7})$$

holds in every period. In this paper we exclude cycles, i.e. equilibria in which (A.7) holds at every period and the signs of its two terms alternate with some specific frequency. Hence, (A.7) holds at every date either because both terms are zero, or one of these terms is positive, and the other one is negative (with the same absolute value) at every date. We next show that the latter case cannot be an equilibrium. Consider first the case in which $\psi_{+1} > \psi$ in every period. This is not consistent with equilibrium unless the sequence $\{\psi_t\}_{t=0}^{\infty}$ converges asymptotically to a finite real number. However, at steady state (13) implies that $\psi_{+1} = -R + [\beta e(q^S)]^{-1}\psi$, i.e. the evolution of the price of the asset is given by a linear first-order difference equation. Hence, if $\psi_{+1} > \psi$, the sequence $\{\psi_t\}_{t=0}^{\infty}$ is unbounded and equilibrium collapses.

Using a similar argument it can be shown that if $\psi_{+1} < \psi$ at every date, the sequence $\{(\phi M)_t\}_{t=0}^{\infty}$ grows without bound. We conclude that for the class of equilibria under consideration, ψ is constant across time and $\phi - (1 + \mu)\phi_{+1} = 0$, or equivalently $\phi/\phi_{+1} = 1 + \mu$. \square

Existence and uniqueness of steady state equilibria. We consider both the case with only financial assets and the case in which we introduce money.

- *Model without money.* A steady state equilibrium satisfies

$$G(q^S) = 0, \quad (\text{A.8})$$

where $G(q) \equiv -z(q) + \beta z(q)e(q) + RT$. Since $G(0) > 0$, a sufficient condition for the existence of a steady state $q^S \in (0, \bar{q}]$ is $G(\bar{q}) \leq 0$. This condition reduces to

$$T \leq \frac{(1 - \beta)z(\bar{q})}{R} \equiv \bar{T}. \quad (\text{A.9})$$

We now show that (A.9) is also necessary for the existence of a steady state equilibrium with $q^S \leq \bar{q}$ and $a_{+1} = 0$. To see why this is true, suppose that an equilibrium of the class described above exists. Equation (7) implies that $(R + \psi)T = z(q^S)$ holds at every date. Leading this equation by one period, multiplying it by β , and subtracting it from the original equation yields

$$T[\psi - \beta(R + \psi_{+1})] = (1 - \beta)z(q^S) - RT.$$

Lemma 3 implies that the left-hand side of the expression above is non-negative. Therefore, a necessary condition for the existence of a steady state equilibrium with $q^S \leq \bar{q}$ and $a_{+1} = 0$ is $T \leq \bar{T}$.

Next, we establish uniqueness of equilibrium under the additional assumption that $e'(q) < 0$ for all $q \in (0, q^*)$. Equation (7) implies that the steady state equilibrium prices satisfy $\psi = \psi_{+1}$ at every date. Using this result, the steady state quantity and the price of the asset can be determined by solving the following system of equations

$$\psi = -R + \frac{z(q^S)}{T}, \quad (\text{A.10})$$

$$\psi = \frac{\beta}{1 - \beta e(q^S)} Re(q^S). \quad (\text{A.11})$$

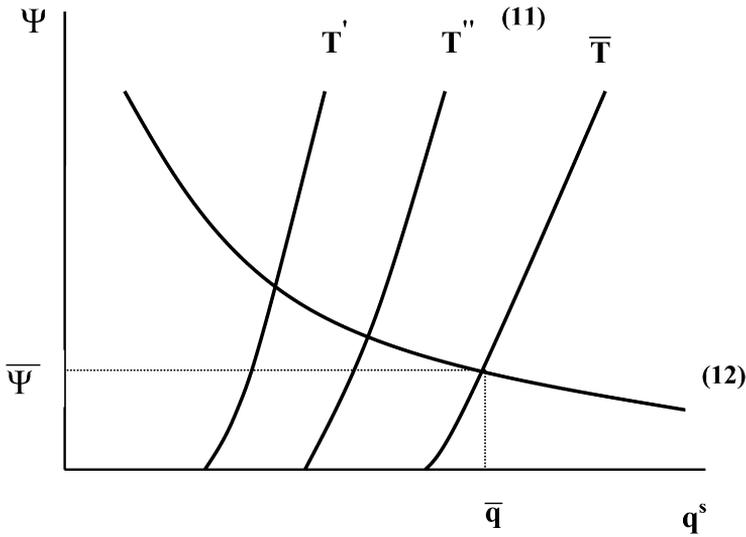


Fig. 2. Equilibria for $T' < T'' < \bar{T}$.

Equation (A.10) defines ψ as a strictly increasing function of q^S . In addition, (A.11) implies that $\frac{d\psi}{dq^S} = \frac{\beta R e'(q^S)}{[1 - \beta e(q^S)]^2}$, which is negative if $e'(q) < 0$. We conclude that, if $T \leq \bar{T}$, there exists a unique steady state equilibrium with $a_{+1} = 0$ and $q^S \leq \bar{q}$ (and $q^S = \bar{q}$ only if $T = \bar{T}$). For $T < \bar{T}$, it can be easily verified that q^S is strictly increasing in β, ξ and θ .²² Finally, notice that T appears only in (A.10). Therefore, a shift in T results in a higher q^S and a lower ψ , as shown in Fig. 2.

Next consider $T > \bar{T}$. We show that for every T in this range there exists a unique equilibrium with $a_{+1} > 0$.²³ Lemma 4 suggests that the agent chooses $a_{+1} > 0$ only if prices satisfy $\psi = \beta(R + \psi_{+1})$. This implies that the price of the asset in such an equilibrium evolves according to the simple difference equation $\psi_{+1} = -R + (1/\beta)\psi$. It can be easily verified that the only bounded solution to this equation satisfies $\psi = \bar{\psi} \equiv [\beta/(1 - \beta)]R$ at every date.²⁴ To conclude the argument, it suffices to show that, for $\psi = \bar{\psi}$, the real balances exceed $z(\bar{q})$. This condition is needed to justify the claim that in this equilibrium $a_{+1} > 0$. In fact, we have

$$(R + \bar{\psi})T > \left(R + \frac{\beta}{1 - \beta}R\right) \frac{(1 - \beta)z(\bar{q})}{R} = z(\bar{q}).$$

²² This result relies on the fact that $e'(q^S) < 0$, and it is valid even if we do not assume that $e'(q) < 0$ for all $q \in (0, q^*)$. The second-order condition of the maximization problem states that, at the optimum, $J_{bb}(\omega_{+1}) < 0$. By Lemma 2, the signs of the terms $J_{bb}(\omega_{+1}) < 0$ and $e'(q)$ coincide. Hence, we conclude that in any equilibrium $e'(q) < 0$.

²³ Since $T \leq \bar{T}$ was shown to be a necessary condition for the existence of an equilibrium with $a_{+1} = 0$, the only possible equilibrium here has $a_{+1} > 0$.

²⁴ This result is consistent with (A.11). For T in the range under consideration we have $q^S = \bar{q}$, and so $e(q^S) = 1$. Hence, the price of the asset implied by (A.11) for $q^S = \bar{q}$ coincides with the formula given above.

- *Money and real assets.* Recall that in any steady state equilibrium with $a_{+1} = 0$, $\phi M + (R + \psi)T = z(q^S)$ holds at every date. Leading this equation by one period, multiplying it by β , and subtracting it from the original equation implies

$$M[\phi - \beta\phi_{+1}(1 + \mu)] + T[\psi - \beta(R + \psi_{+1})] = (1 - \beta)z(q^S) - RT,$$

which, utilizing (14), can be rewritten as

$$M\phi(1 - \beta) + T[\psi - \beta(R + \psi_{+1})] = (1 - \beta)z(q^S) - RT.$$

Both terms on the left-hand side of this expression are non-negative. Hence, a necessary condition for the existence of a steady state equilibrium with $a_{+1} = 0$ is

$$T \leq [(1 - \beta)z(q^S)]/R \leq \bar{T},$$

where \bar{T} is defined in (A.9) and the last inequality follows from the fact that z is strictly increasing in q .

If $T > \bar{T}$, the only possible equilibrium has $a_{+1} > 0$. As in the previous section, we guess and verify that such an equilibrium exists for every T in the relevant region. Since the candidate equilibrium has $a_{+1} > 0$, optimality requires $\psi = \beta(R + \psi_{+1})$, and the sequence $\{\psi_t\}_{t=0}^{\infty}$ is bounded only if $\psi = \bar{\psi}$ in every period, where $\bar{\psi} \equiv [\beta/(1 - \beta)]R$ as in Section 3. It remains to show that, for $\psi = \bar{\psi}$, the total real balances exceed $z(\bar{q})$. We have

$$\phi M + (R + \bar{\psi})T \geq \left(R + \frac{\beta}{1 - \beta}R\right)T > \left(R + \frac{\beta}{1 - \beta}R\right)\frac{(1 - \beta)z(\bar{q})}{R} = z(\bar{q}),$$

which concludes the argument. \square

Feasible monetary policies. For any given $T < \bar{T}$, define ψ^T and $q^{S,T}$ as the (unique) steady state solution for the economy without money. As we have already seen, $q^{S,T} < \bar{q}$ and $\psi^T > \bar{\psi}$. Next, define $\mu^T \equiv \{\mu : \frac{1+\mu}{-\mu}R = \psi^T\}$.²⁵ It follows from (17) that this value of μ is uniquely defined, and that for all $T < \bar{T}$, $\mu^T < 0$ and $d\mu^T/dT < 0$. Then, consider any $\mu_1 \in (\mu^T, 0)$, and let ψ_1 and q_1^S be the equilibrium values determined through (17) and (15), respectively. Since $\mu_1 > \mu^T$, we have $\psi_1 > \psi^T$ and $q_1^S < q^{S,T}$. In any steady state equilibrium $\phi M + (R + \psi)T = z(q^S)$. This implies that following the policy rule μ_1 would lead to $\phi M < 0$, which is not consistent with equilibrium. \square

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²⁵ Clearly, μ^T is a neutral monetary policy, in the sense that it yields the same equilibrium quantity and asset price as in the economy without money.

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