Heterogeneous Asset Valuation in OTC Markets and Optimal Inflation

Athanasios Geromichalos*
University of California, Davis

Kuk Mo Jung†
Sogang University

November 28, 2023

Abstract

Building on recent work in monetary theory and finance, we develop a framework where money serves a *double liquidity role*, namely, it serves as a medium of exchange in goods markets as well as asset markets. We argue that studying such a framework is not only more empirically relevant, but also gives rise to new, important economic insights regarding the effects of inflation on welfare and asset prices. The main result of the paper is that, contrary to conventional wisdom, in our model welfare can be increasing in inflation due to a new channel whereby higher inflation promotes beneficial trade in the secondary asset market.

Keywords: monetary-search models, liquidity, asset prices, over-the-counter markets, inflation-welfare relationship

JEL Classification Numbers: E31, E50, E52, G12

*Address: One Shields Avenue, Davis, CA 95616, Phone: 530 752 9241, E-mail: *ageromich@ucdavis.edu*

†Address: School of Economics, Sogang University, 35 Baekbeom-Ro (Sinsu-Dong) Mapo-Gu, Seoul 04107, Republic of Korea., E-mail: *kmjung@sogang.ac.kr*.

1 Introduction

The welfare cost of inflation is one of the most important and extensively studied questions in monetary theory. An almost universal result that arises in this literature is that an increase in inflation will hurt welfare: inflation acts as a tax on real balances, hence, an increase in this tax reduces agents' real money holdings which, consequently, reduces the quantity of goods they can purchase.[1] A common thread running through the majority of papers that predict such a negative relationship between inflation and welfare is that they adopt models where money serves as a medium of exchange exclusively in *goods/product markets*.[2] However, a more recent strand of the literature, initiated by the seminal work of Lagos and Zhang (2020a), highlights that money also serves as a medium of exchange in *asset markets*, and that this role of money can be crucial for the transmission of monetary policy on asset prices and welfare.

The objective of this paper is to develop a theoretical framework where money plays a double liquidity role: it serves as a medium of exchange in a goods market, as is standard in traditional monetary theory, but it also serves as a medium of exchange in an over-the-counter (OTC) financial market, as is the case in the more recent work of Lagos and Zhang (2019, 2020a). To the best of our knowledge, this is the first attempt to explicitly model this *double liquidity role* of money. Our goal is to highlight that building such a framework (is not just more empirically relevant, but also) offers new and important economic insights regarding the effects of inflation on welfare and asset prices. The main result of the paper is that in our model welfare can be increasing in inflation, contrary to the majority of the literature, where a negative relationship between inflation and welfare is typically established.

To understand the mechanism behind this new result, we need to describe the basic structure of our model. We build on the framework of Lagos and Wright (2005), but we extend it to allow agents to hold a portfolio of money *and* assets. To generate gains from trade in the OTC market, we follow Duffie, Gârleanu, and Pedersen (2005), and we assume that some agents have a low valuation while other agents have a high valuation for the (same) asset. Thus, in our environment, money is useful to purchase goods in a decentralized product market, but also to purchase assets in the OTC market *if* they turn out to be high-valuation types, because these types have an incentive to purchase assets from low-valuation types. Consequently, changes in inflation affect welfare through the traditional "real money balances channel", but also through a new channel which captures agents' willingness to trade assets for money in the secondary OTC market.

[1] However, exceptions to this rule do exist and we report them in detail in the literature review.

[2] See, for example, Lagos and Wright (2005) and the large literature that builds on their framework.

Given the description of the modeling environment, we can now understand the mechanism that gives rise to our main result. As in any other model that builds on the Lagos and Wright (2005) framework, here too, a higher inflation has a negative effect on the equilibrium real balances agents choose to carry. This tends to decrease the amount of trade in the goods market and hurt welfare. However, in our model, agents get a chance to rebalance their money holdings by trading assets in the OTC market. A higher inflation means that agents brought lower money balances initially, this means that agents are further away from the first-best level of consumption, which, in turn, means that low-valuation types are more willing to sell assets to high-valuation types and boost their money holdings. Through this new channel a higher inflation tends to increase the volume of trade in the OTC market, which tends to increase welfare because in our framework optimality requires that the asset be concentrated in the hands of high-valuation types.

The discussion so far does not establish that an increase in inflation will be welfare improving; it only illustrates a new channel whereby higher inflation "tends" to improve welfare. Inflation will be welfare improving only if the newly established positive effect is larger than the traditional, negative real balance effect. Our earlier discussion reveals that the potential welfare gains from a higher inflation are *linear* because the asset valuation of the different types is constant.[3] On the other hand, the welfare loss due to the lower real balances, and the resulting reduction in consumption, manifests itself as a move along a *concave* utility function. Quite naturally, it turns out that the conditions under which an inflation increase can become welfare improving are closely related to the *curvature* of the utility function of the decentralized market good. Focusing on the CRRA family of utility functions, we find that inflation can be welfare improving if the risk aversion parameter is greater than 1.

Intuitively, a high curvature of the utility function implies that low-valuation types have a lot to win, i.e., a high marginal utility, by acquiring more cash, which allows them to purchase more good and get closer to their first-best consumption level. It also implies that low-valuation types are more willing to sell assets to the high-valuation types to acquire that cash. But this, i.e., a transfer of assets from low- to high-valuation types, is precisely what generates the aforementioned new, positive channel in our model. Thus, a higher risk aversion parameter (equivalently, a higher curvature of the utility function) is more likely to make welfare increasing in inflation. It should be noted that this condition is necessary but not sufficient for the main result. Additionally, we need a relatively large asset supply

[3] Simply put, high types get a higher yield/dividend by holding the asset, but that yield does not depend on the amount of assets they hold. More precisely, let α_j, $j \in \{L, H\}$, with $\alpha_H > \alpha_L$, denote the asset valuations for each type. Then, every time an amount χ of assets changes hands in the OTC, an additional welfare equal to $\chi(\alpha_H - \alpha_L)$ is generated. See Section 2 for more details.

and high inflation. This last detail reveals another important result: in our model there exist regions in which inflation is welfare improving, but the optimal level of inflation is the Friedman rule.

Our model also allows us to study the effects of inflation on OTC trade volume and the price of the asset in that market. A higher inflation can change OTC trade volume under a relatively large asset supply. Importantly, the volume can change into either direction in our model due to two offsetting forces. First, a higher inflation tightens asset buyers' budget in OTC trade since buyers have lower cash holdings when they enter the OTC market. This causes less OTC asset transfers, namely, a negative effect of inflation on OTC trade volume. On the other hand, a higher inflation increases asset sellers' willingness to acquire money (or willingness to give up assets for a given amount of money). This gives rise to a positive effect of inflation on OTC trade volume. We show that the positive effect dominates if the risk aversion parameter of the CRRA utility function exceeds 1. This is intuitive because a higher curvature implies a higher liquidity value of cash in goods markets. In sum, a higher inflation can stimulate more OTC asset transfer under two conditions: a relatively large asset supply and a relatively high degree of risk aversion. The real asset price in the OTC market tends to decrease in inflation in our model. This is straightforward too. Under the aforementioned two conditions a higher inflation will induce agents to always transfer less real money balances and more assets in OTC markets.

The present paper is related to the recent literature in monetary economics, studying the *indirect* liquidity of assets, i.e., the concept that agents can sell assets for money in a secondary asset market. Examples of such papers include Berentsen, Huber, and Marchesiani (2014), Mattesini and Nosal (2015), Geromichalos and Herrenbrueck (2016), Geromichalos and Jung (2018), and Madison (2019).[4] In these papers, agents receive an (ex post) idiosyncratic consumption shock that determines whether they will be active buyers in the decentralized goods market. Agents with a consumption opportunity seek for agents without such an opportunity and sell assets to them in order to acquire the cash that the latter will not be needing in the current period. In other words, in the "indirect liquidity" literature, money is the medium of exchange *only* in the goods market, and agents who find out that they need it can boost their money balances by selling assets in the OTC market. However, all agents have the same valuation for assets, so trading assets is just the means for acquiring more money, not the goal per se. Here, there is a *fundamental* reason for agents to trade in

[4] Another strand of the literature studies asset liquidity in environments where assets have "direct" liquidity properties, i.e., they serve directly as means of payment; see for example Geromichalos, Licari, and Suarez-Lledo (2007), Lagos (2010), Nosal and Rocheteau (2013), Andolfatto, Berentsen, and Waller (2014), Hu and Rocheteau (2014), Han, Julien, Petursdottir, and Wang (2019), Jung and Pyun (2016), and Lee and Jung (2020).

the OTC market: they have different valuations for the asset. Thus, in our model money is a medium of exchange both in the goods market and the asset market, and agents must make their money holding decisions balancing out these two roles.[5]

Our work is also related to the literature initiated by Duffie et al. (2005), which studies how frictions in OTC markets affect asset prices and trade; examples include Weill (2007), Lagos and Rocheteau (2009), Chang and Zhang (2015), Üslü (2019), and Gabrovski and Kospentaris (2021). In these papers, like in ours, agents trade in OTC markets because they have a different valuation for the asset. However, none of these papers explicitly models the idea that agents need a medium of exchange in order to purchase assets, which, of course, also implies that agents may be liquidity constrained. (In these papers, agents can basically buy any amount of assets they wish using transferable utility.) The recent work of Lagos and Zhang (2019, 2020a) extends the framework of Duffie et al. (2005) by explicitly modeling the exchange process, i.e., the idea that agents need money to purchase assets, but these papers do not incorporate a goods market. Thus, to the best of our knowledge, this is the first paper that explicitly models money as a medium of exchange in goods *and* asset markets concurrently. As argued earlier, exploring this *double role* of money offers important new economic insights.

Finally, our paper is related to a large literature in monetary economics studying the effect of inflation on welfare. As already discussed, a prevailing result in this literature is that higher inflation amounts to a higher tax on real balances, which decreases the amount of goods agents can purchase, thus, hurting welfare. There are two classes of models that present exceptions to this general rule. (For an exhaustive list, see Section 6.9 of Nosal and Rocheteau (2011)). The first class is models where inflation can increase welfare through distributive effects; see for example Molico (2006) and Rocheteau, Weill, and Wong (2015). The second class is monetary-search models where agents make entry decisions in the goods market; see for example Rocheteau and Wright (2005) and Berentsen, Rocheteau, and Shi (2007). In the latter, the Friedman rule achieves efficiency at the intensive margin, but an increase in inflation can be welfare improving by increasing efficiency at the extensive margin. In related work, Geromichalos and Jung (2019) offer a model where higher inflation can lead to welfare improvement by affecting asset (as opposed to goods) market participation decisions. Also, Wright, Xiao, and Zhu (2020) extend the Lagos and Zhang (2020a) framework by considering the case of capital, as opposed to a Lucas tree. The authors show that the optimal inflation will exceed the Friedman rule if the bargaining power of buyers in the secondary market for

[5] For instance, if an agent brings a limited amount of money and they use most of it to purchase assets in the OTC market, they realize that this will affect their ability to purchase goods in the product market. This type of trade off must be taken under consideration when agents make their portfolio decisions.

capital is too large; this is because a high buyer-bargaining power in the secondary market will lead to underinvestment in the primary market, and a higher inflation can correct this inefficiency. Our model is distinguished from all this work, as it studies the role of money as a medium of exchange in goods *and* asset markets concurrently, and shows that higher inflation can improve welfare by promoting beneficial asset trade in secondary markets.

2 Model

Time is discrete and infinite. Within each period there exist three sub-periods where different economic activities take place. During the first sub-period, a financial market opens, which resembles the OTC market of Duffie et al. (2005). We refer to this market as the OTC market. In the second sub-period, agents trade for goods in a pairwise fashion as in Lagos and Wright (2005). We refer to this as the LW market. In the third sub-period, economic activity takes place in a frictionless centralized market. This market can be thought of as a settlement market, and we refer to it as the CM. A detailed description of these markets will be provided below. There are two types of economic agents, consumers and producers, depending on their role in the LW market. Agent types are permanent by assumption (i.e., as in Rocheteau and Wright (2005)). The measure of both types is normalized to the unit.

The discount rate is given by $\beta \in (0, 1)$. Agents discount between periods but not between sub-periods. Consumers consume in the second and the third sub-periods and supply labor in the third sub-period. Their preferences are given by $\mathcal{U}(X, H, q)$, where X, H stand for consumption and labor in the CM, respectively, and q is consumption in the LW market. The typical consumer's LW utility is give by $u(q)$. Producers produce in both the CM and the LW market, but consume only in the CM. Their preferences are described by $\mathcal{V}(X, H, q)$, where X, H are as above, and q represents units of the LW good produced. Functional forms for consumers and producers are given by $\mathcal{U}(X, H, q) = X - H + u(q)$, and $\mathcal{V}(X, H, h) = X - H - c(q)$. We assume that u is twice continuously differentiable with $u(0) = 0$, $u' > 0$, $u'(0) = \infty$, $u'(\infty) = 0$. For simplicity, we set $c(q) = q$, but this is not crucial for any results. Let q^* denote the optimal level of production in a meeting between a consumer and a producer in the LW market, i.e., q^* solves $u'(q^*) = 1$.

In the third sub-period, a *general* good is produced and consumed by all agents. This good also plays the role of the numeraire in our model. Agents have access to a technology that transforms one unit of labor into one unit of the general good. Following, Mattesini and Nosal (2015), we assume that in the third sub-period of each date, t, each consumer is endowed with A units of a real asset. This asset cannot be traded in the third sub-period; agents must proceed into the next period holding the amount they were endowed with. Each unit of the asset delivers a dividend, in terms of the general good, in the CM of $t+1$ and then it "dies". What is new here is that after consumers have entered the new period, they are subject to an idiosyncratic shock that affects their valuation of the (same) asset. Some agents, who we will refer to as the H-types, obtain a dividend equal to α_H, while others, referred to as the L-types, obtain a dividend equal to α_L, with $\alpha_H > \alpha_L > 0$. This idiosyncratic preference shock is realized at the beginning of each period, and it is *i.i.d.* across periods and agents. Following Duffie et al. (2005), and the large literature spurred by this paper, we think of the L-types as agents who have high(er) financing costs, hedging reasons to sell, a relative tax disadvantage, or, simply, a lower personal use of the asset. Hence, in our model, trade in the OTC market takes place because agents have a different (*ex post*) valuation for the same asset; specifically L-types wish to sell assets to H-types. This is in contrast to the "indirect liquidity" models mentioned in Section 1, where agents trade in secondary asset markets because they receive an idiosyncratic consumption shock.

In our economy there is also a second asset, namely, fiat money. Money is traded in the CM, and we let φ_t denote its price (which agents take as given). Standard assumptions apply. Money supply is controlled by a monetary authority and evolves according to $M_{t+1} = (1+\mu)M_t$, with $\mu > \beta - 1$. New money is introduced, or withdrawn if $\mu < 0$, via lump-sum transfers to consumers in the CM. Money is durable, divisible, and recognizable by all agents, i.e., it possesses all the properties that make it appropriate to serve as a medium of exchange in the LW (goods) market.[6] As we have already mentioned, one of the unique contributions of our paper is to explicitly model the idea that money also serves as a medium of exchange in the OTC asset market.

In our framework, it is the consumers who make all the interesting economic decisions. (As it is shown in Rocheteau and Wright (2005), producers in these types of models will typically not want to leave the CM with positive money holdings.) Thus, hereafter we refer to consumers simply as "agents".

We now explain trading activities in the OTC market. After leaving the CM agents receive the aforementioned idiosyncratic shock. A measure $\ell < 1$ of agents become H-types, and the remaining $1-\ell$ agents will be L-types. Once the shocks have been revealed, L-types realize they will value the dividend less than H-types in the forthcoming CM, and this gives rise to potential gains from trade between the two types in the OTC market. Thus, L-types enter the OTC market as liquidity providers/asset sellers, and H-types enter that market as liquidity seekers/asset buyers. OTC market trades take place bilaterally. Agents must

[6] We assume that the real asset cannot serve as a medium of exchange in the goods market. For a discussion on the possible micro-foundations behind this assumption, see Rocheteau (2011), Lester, Postlewaite, and Wright (2012), and Geromichalos, Jung, Lee, and Carlos (2021).

search for their counterparts, and search frictions preclude a perfect matching by assumption. We assume that the total measure of matches equals $f(\ell, 1-\ell) \leq min\{\ell, 1-\ell\}$, where f represents a matching function which brings H-types and L-types together. Once pairs are formed, the involved parties bargain over the quantity of assets to be transferred from the asset seller (L-type) to the asset buyer (H-type). Money will serve as a sole medium of exchange during this negotiation. For the sake of simplicity we assume H-type makes a *take-it-or-leave-it* offer to L-type within each pairwise meeting.

Once OTC trade has concluded, all agents proceed to the LW market operating within the second sub-period. This is the standard decentralized market of Lagos and Wright (2005). L-type and H-type agents meet with producers in a bilateral fashion and negotiate over the terms of trade. As in standard search models, anonymity and imperfect commitment lead to *quid pro quo* exchange and, as in the preceding OTC market, only money can serve as means of payment. In this framework, all the interesting results emerge from agents' interaction in the OTC market. Again for the sake of simplicity, we assume that all agents match with a producer, and they make a take-it-or-leave-it (TIOLI) offer.

3 Value Functions

We begin with the description of the value functions in the CM. Throughout the paper we drop time subscripts, and we let variables with "hats" denote values associates with next period. For example, $\hat{m}$ denotes the agent's money holdings for next period, and $\hat{\varphi}$ denotes the real price of money in the next period, and so on. Consider a type $j \in \{H, L\}$ agent who enters the CM with money and asset holdings $(m, a) \in \mathbb{R}^2_+$. For this agent the CM value function is given by

$$W_j\left(m, a\right) = \max_{X, H, \hat{m}} \left\{X - H + \beta \mathbb{E}\left\{\Omega(\hat{m})\right\}\right\}$$
$$\text{s.t. } X + \varphi \hat{m} = H + \varphi(m + \mu M) + \alpha_j a,$$

where $\alpha_j \in (\alpha_H, \alpha_L)$ represents the idiosyncratic dividend shock for a type j agent, and, as we have discussed, $\alpha_H > \alpha_L$. $\Omega(\hat{m})$ represents OTC value function. Replacing for the agent's net consumption, $X - H$, from the budget constraint into the objective function gives us

$$W_j(m, a) = \varphi m + \alpha_j a + \Upsilon, \tag{1}$$

where Υ equals $\varphi\mu M + \max_{\hat{m}} \{-\varphi\hat{m} + \beta\mathbb{E}\{\Omega(\hat{m})\}\}$.

Next, consider the CM value function for a producer.

$$W^P(m) = \varphi m + \beta V^P \equiv \varphi m + \Upsilon^P, \tag{2}$$

where V^P denotes the producer's value function in the next period's LW market.

After leaving the CM, and before the OTC opens, agents are endowed with assets A, and learn their type $j = \{L, H\}$. Therefore, the expected value for an agent who carries m units of money before she enters the OTC market is given by

$$\mathbb{E}\{\Omega(m)\} = \ell\, \Omega_H(m) + (1-\ell)\, \Omega_L(m), \tag{3}$$

where ℓ is the measure of H-type agents. In any OTC meeting between an asset buyer (H-type) and asset seller (L-type), let $\chi \geq 0$ denote the units of assets that the L type transfers to the H type. Let $\delta \geq 0$ denote units of money that the H type transfers to the L type. These terms will be determined through bargaining later. We have:

$$\Omega_L(m) = \pi_L V_L(m+\delta, A-\chi) + (1-\pi_H) V_L(m, A), \tag{4}$$

$$\Omega_H(m) = \pi_H V_H(m-\delta, A+\chi) + (1-\pi_H) V_H(m, A), \tag{5}$$

where the probability, π_j, is the probability that the j-type gets matched with $-j$ type and V_j where $j = \{L, H\}$, denotes the j-type's value function in the LW market. Note that $\pi_H = f(\ell, 1-\ell)/\ell$ and $\pi_L = f(\ell, 1-\ell)/(1-\ell)$.

Lastly, consider the value functions in the LW market. Let q_j denote the quantity of good produced for the j-type agent, and p_j the payment, in monetary units, made by the j-type agent to the producer. The LW market value function for the j-type with portfolio (m, a) is given by

$$V_j(m, a) = u(q_j) + W_j(m - p_j, a). \tag{6}$$

The LW value function for a producer (who enters with no money or assets) is simply $V^P = -q_j + W^P(p_j)$.

We can now proceed to the description of the terms of trade in the LW and OTC markets. We start with the easier LW market bargaining problem. Consider a meeting between a producer and an j-type agent, $j = \{L, H\}$, with portfolio (m, a). The two parties bargain over the quantity, q_j, and the total monetary payment, p_j, and the j-type agent makes a TIOLI offer, maximizing her surplus subject to the producer's participation constraint and the cash constraint. Hence, the bargaining problem is given by

$$\max_{p_j,q_j} \left\{u(q_j) + W_j(m - p_j, a) - W_j(m, a)\right\},$$

subject to $-q_j + W^P(p_j) - W^P(0) = 0$, and $p_j \leq m$. Substituting the value functions W_j and W^P from (1) and (2) into these expressions simplifies the bargaining problem to

$$\max_{p,q} \left\{u(q) - \varphi p\right\},$$

subject to $q = \varphi p$, and $p \leq m$. Note that the LW bargaining problem is identical to both types of agents. The solution to this bargaining problem is as follows.

Lemma 1 *Define the amount of money that, given the price φ, allows the type-j agent to purchase q^* as $m^* \equiv q^*/\varphi$. Then, the solution to the bargaining problem is given by $q(m) = \min\{\varphi m, q^*\}$ and $p(m) = \min\{m, m^*\}$.*

Proof. This result is standard in these types of models, and therefore omitted. ■

Next, consider a meeting in the OTC market where an asset buyer, i.e., a type-H agent, with money holdings m, makes a *take-it-or-leave-it* (TIOLI) offer to an asset seller, i.e., a type-L agent, with money holdings $\tilde{m}$ (recall that both of these agents carry A units of the asset as they enter the OTC). Exploiting (6) and Lemma 1, the OTC bargaining problem can be expressed as

$$\begin{aligned} &\max_{\delta,\chi} \left\{u\left(q(m-\delta)\right) - u\left(q(m)\right) + \alpha_{H}\chi - \varphi\delta + \varphi\left[p(m) - p(m-\delta)\right]\right\} \\ &\text{s.t. } u\left(q(\tilde{m}+\delta)\right) - u\left(q(\tilde{m})\right) - \alpha_{L}\chi + \varphi\delta - \varphi\left[p(\tilde{m}+\delta) - p(\tilde{m})\right] = 0, \\ &\qquad \chi \leq A, \\ &\qquad \delta \leq m, \end{aligned}$$

where $\{q(.), p(.)\}$ are described in Lemma 1. Lemma 2 describes the bargaining solution.

Lemma 2 *Define $\bar{m}$, $\Lambda(m,\tilde{m})$, and $\lambda(\tilde{m})$ respectively as $\bar{m} = \{m : u'(\varphi m) = \alpha_{H}/\alpha_{L}\}$,*

$$\Lambda(m,\tilde{m}) = \begin{cases} \{\Lambda : u'(q(m-\Lambda)) = \alpha_{H}/\alpha_{L}\} & \text{if } m+\tilde{m} \geq m+\bar{m}, \\ \{\Lambda : u'(q(m-\Lambda)) = (\alpha_{H}/\alpha_{L})u'(q(\tilde{m}+\Lambda))\} & \text{if } m+\tilde{m} < m+\bar{m}, \end{cases}$$

$$\lambda(\tilde{m}) = \begin{cases} \{\lambda : \alpha_{L}A = u(q^*) - q^* - u(q(\tilde{m})) + \varphi\left(p(\tilde{m}) + \lambda\right)\} & \text{if } m+\tilde{m} \geq m+\bar{m}, \\ \{\lambda : \alpha_{L}A = u(q(\tilde{m}+\lambda)) - u(q(\tilde{m}))\} & \text{if } m+\tilde{m} < m+\bar{m}. \end{cases}$$

Also, define the cutoff level of asset holdings as below.

$$\bar{a}(m,\tilde{m}) = \begin{cases} \frac{u(q^*)-q^*-u(q(\tilde{m}))+\varphi(p(\tilde{m})+\Lambda(m,\tilde{m}))}{\alpha_L} & \text{if } m+\tilde{m} \geq m+\bar{m}, \\ \frac{u(q(\tilde{m}+\Lambda(m,\tilde{m})))-u(q(\tilde{m}))}{\alpha_L} & \text{if } m+\tilde{m} < m+\bar{m}. \end{cases}$$

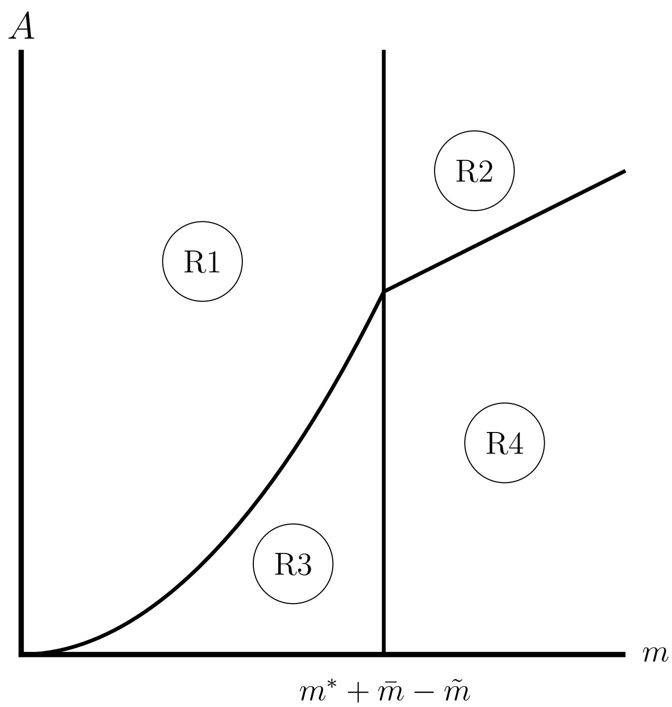
Then the solution to the bargaining problem is given by

$$\chi(m,\tilde{m}) = \begin{cases} \bar{a}(m,\tilde{m}) & \text{if } A \geq \bar{a}(m,\tilde{m}), \\ A & \text{if } A < \bar{a}(m,\tilde{m}), \end{cases}$$

$$\delta(m,\tilde{m}) = \begin{cases} \Lambda(m,\tilde{m}) & \text{if } A \geq \bar{a}(m,\tilde{m}), \\ \lambda(\tilde{m}) & \text{if } A < \bar{a}(m,\tilde{m}), \end{cases}$$

Proof. See Appendix A.3. ■

Figure 1: OTC Bargaining Solutions



There are 4 regions for OTC bargaining solutions.

1. R1: plentiful asset and scarce money where $m - \delta < m^*$, $\tilde{m} + \delta < m^*$, and $A > \chi$.

2. R2: plentiful asset and plentiful money where $m - \delta < m^*$, $\tilde{m} + \delta \geq m^*$, and $A > \chi$.

3. R3: scarce asset and scarce money where $m - \delta \leq m^*$, $\tilde{m} + \delta < m^*$, and $A = \chi$.

4. R4: scarce asset and plentiful money where $m - \delta \leq m^*$, $\tilde{m} + \delta \geq m^*$, and $A = \chi$.

In principle, the H-type will like to buy up the entire amount of A from the L-type. This is actually what happens in Region 3 and 4. The H-type gives up δ units of money such that the L-type's asset loss in utility terms $\alpha_{_L} A$ is compensated by the LW utility gains, e.g., $u(q(\tilde{m} + \delta)) - u(q(\tilde{m}))$ in Region 3.

However, the H-type cannot always buy up the entire A when the latter gets larger. Intuitively, she buys assets up to the point where marginal benefit from giving up 1 unit of real money, i.e., CM dividend utility ($\alpha_{_H}$) times marginal units of assets received from the L-type ($u'(q(\tilde{m} + \delta))/\alpha_{_L}$), equals marginal cost of giving up 1 unit of real money, i.e., $u'(q(m - \delta))$ in Region 1. This marginal cost is increasing in the amount of money the H-type gives up. The term $\bar{a}(m, \tilde{m})$ represents the threshold where the marginal benefit equals the marginal cost. If A gets bigger than that threshold level, she will not buy more assets because the marginal cost will exceed the marginal benefit then.

3.1 Objective Function and Optimal Behavior

To obtain the objective function, substitute (4) and (5) into (3), and lead the emerging expression for $\mathbb{E}\{\Omega\}$ by one period. Next, substitute the value functions W and V_j from (1) and (6) into this expression. Finally, substitute the resulting expression for $\mathbb{E}\{\Omega(\hat{m})\}$ into (1), and focus only on the terms that are relevant to the agent's control variable, $(\hat{m})$. After some algebra, one can verify that the objective function, J, is given by:

$$
\begin{aligned}
J(\hat{m}) = &-(\varphi - \beta\hat{\varphi})\,\hat{m} \\
&+ \beta f\big\{u\,(q(\hat{m} - \delta)) + \alpha_{_H}(A + \chi) - \hat{\varphi}[\delta + p(\hat{m} - \delta)]\big\} \\
&+ \beta(\ell - f)\big\{u\,(q(\hat{m})) + \alpha_{_H} A - \hat{\varphi} p(\hat{m})]\big\} \\
&+ \beta f\big\{u(q(\hat{m} + \tilde{\delta})) + \alpha_{_L}(A - \tilde{\chi}) + \hat{\varphi}[\tilde{\delta} - p(\hat{m} + \tilde{\delta})]\big\} \\
&+ \beta(1 - \ell - f)\big\{u(q(\hat{m})) + \alpha_{_L} A - \hat{\varphi} p(\hat{m})\big\}.
\end{aligned} \tag{7}
$$

The interpretation of J is intuitive. It is understood that the various expressions χ, δ are determined in Lemma 2, and we have $\chi = \chi(\hat{m}, \ddot{m})$, $\delta = \delta(\hat{m}, \ddot{m})$, and $\tilde{\chi} = \chi(\ddot{m}, \hat{m})$, $\tilde{\delta} = \delta(\ddot{m}, \hat{m})$, where $\ddot{m}$ is the agent's expectation about the money holdings of the agent that she will encounter in the OTC market.[7]

[7] Recall that Lemma 2 describes the various χ, δ terms as functions of the vector $(m, \tilde{m})$, where m is the $H - type$ agent's money holdings and $\tilde{m}$ is the L-type agent's money holdings. Also, notice that the terms $\tilde{\chi}, \tilde{\delta}$ in the objective function refer to the case in which the agent is a L type. This is precisely why in these

We relegate technical details of the representative agent's optimal choice of $\hat{m}$ to Appendix A.1 and turn to general equilibrium.

4 Equilibrium

4.1 Definition and Properties of Equilibrium

We restrict attention to symmetric, monetary steady state equilibria, where all agents choose the same positive amount of real money balances, and the real variables of the model remain constant over time. We define Z as the steady state level of real money balances, which does not change over time. Thus, we have $\varphi M = \hat{\varphi}\hat{M}$, implying that $\varphi/\hat{\varphi} = 1 + \mu$. We assume $\mu > \beta - 1$.

Definition 1 *A steady state equilibrium consists of a list of bargaining solutions for the LW and OTC markets, $\{(p_i, q_i), (\chi, \delta)\}$, described in Lemmas 1 and 2, together with a choice of money holdings, $\hat{m}$, and prices, $\{\varphi, \hat{\varphi}\}$, such that:*

- *$\hat{m}$ solves the individual optimization problem (7), taking prices as given.*
- *CM clears and expectations are rational: $\hat{m} = \ddot{m} = (1 + \mu)M$.*
- *Real money balances remain constant over time: $\varphi/\hat{\varphi} = 1 + \mu$.*

Proposition 1 *Assume a CRRA utility, $u(q) = q^{1-\rho}/(1-\rho)$. Let $\bar{\bar{q}} = \{q : u'(q) = q^{-\rho} = \alpha_H/\alpha_L\}$. Also, let η (χ) denotes the real value of money (asset) transfer in OTC at the steady state.*

a) There exists a unique steady state monetary equilibrium, which has 4 aggregate equilibrium regions for any parameterization. The 4 regions have the following properties.

- *Region 1: $Z < (q^* + \bar{\bar{q}})/2$, $Z - \eta < q^*$, $Z + \eta < q^*$ and $A > \chi$.*
- *Region 2: $Z \geq (q^* + \bar{\bar{q}})/2$, $Z - \eta < q^*$, $Z + \eta \geq q^*$ and $A > \chi$.*
- *Region 3: $Z < (q^* + \bar{\bar{q}})/2$, $Z - \eta < q^*$, $Z + \eta < q^*$ and $A = \chi$.*
- *Region 4: $Z \geq (q^* + \bar{\bar{q}})/2$, $Z - \eta < q^*$, $Z + \eta \geq q^*$ and $A = \chi$.*

b) In the equilibrium Region 1

$$\frac{\partial \chi}{\partial Z} = \begin{cases} < 0, & \text{if } \rho > 1, \\ = 0, & \text{if } \rho = 1, \\ > 0, & \text{if } \rho < 1, \end{cases} \quad \frac{\partial^2 \chi}{\partial Z^2} = \begin{cases} > 0, & \text{if } \rho > 1, \\ = 0, & \text{if } \rho = 1, \\ < 0, & \text{if } \rho < 1, \end{cases}$$

two expressions the agent's own money holdings, $\hat{m}$, appear as the second argument.

c) In the equilibrium Region 2

$$\frac{\partial \chi}{\partial Z} = \begin{cases} \frac{1}{\alpha_L} > 0, & \text{if } Z \geq q^*, \\ \frac{1}{\alpha_L}\left(-Z^{-\rho}+2\right) > 0, & \text{if } \frac{q^*+\bar{\bar{q}}}{2} \leq Z < q^* \; and \, \rho \leq 1, \\ \frac{1}{\alpha_L}\left(-Z^{-\rho}+2\right) \lesseqgtr 0, & \text{if } \frac{q^*+\bar{\bar{q}}}{2} \leq Z < q^* \; and \, \rho > 1. \end{cases}$$

Also, $\partial^2\chi/\partial Z^2$ *equals 0 if* $Z \geq q^*$*, and is strictly positive if* $(q^* + \bar{\bar{q}})/2 \leq Z < q^*$*.*

Proof. See Appendix A.3. ■

Based on Proposition 1, Figures 2, 3, and 4 illustrate the three equilibrium regions, not as functions of individual choices $\hat{m}, a$, but as functions of the exogenous asset supply A (which, in equilibrium, equals a), and the real money balances Z.[8] Note that they differ depending on the relative vale of risk aversion parameter ρ, which follows from part b) and c) in Proposition 1.

Take as an example the equilibrium Region 1. The L-type agent gives up *less* assets in OTC as her real money balances increase. This happens only if her marginal valuation for real money balances are relatively big, i.e., $\rho > 1$. This is important for welfare analysis later. Intuitively, the amount of asset transfers by the L-type in OTC equals her LW utility gain due to TIOLI offer. This creates two channels through which her willingness to transfer assets to the H-type is determined. The first channel is a real money balance channel. As Z goes up, her desire for extra cash holdings in the forthcoming LW market weakens. This induces her to give up less assets as Z increases. However, there exists an opposite second channel as well. As Z increases, the H-type can afford to hand over more liquidity. This in turn induces the L-type to transfer more assets to the H-type. It turns out the extent to which agents value marginal increase in cash holdings in the LW market, i.e., the curvature of the LW utility function (ρ), determines which channel prevails in equilibrium. As agents become more desperate for extra cash holdings in the LW market, in the precise sense that $\rho > 1$, the first channel dominates. Therefore, more real money balances will cause less asset transfers in this case.

Figures 5 and 6 illustrate the 4 equilibrium regions as functions of the exogenous asset supply A, and the money growth rate μ. They basically show mirror images of Figure 2 and 4. Suppose A is relatively greater such that $A > A_1$. Also assume $\rho \leq 1$, i.e., Figure 6. Then,

[8] Note that in all Figure 2, 3, and 4

$$A_1 = \alpha_L^{-1}\left(\frac{1+\bar{\bar{q}}}{2}\right)^{-\rho}\left\{\left(\frac{2\lambda}{1+\lambda}\right)^{1-\rho} - 1\right\}, \; A_2 = \frac{1-\lambda^{-1}}{\alpha_L}, \; \lambda = (\alpha_H/\alpha_L)^{1/\rho} > 1.$$

Figure 2: Aggregate regions of equilibrium (if $\rho > 1$)

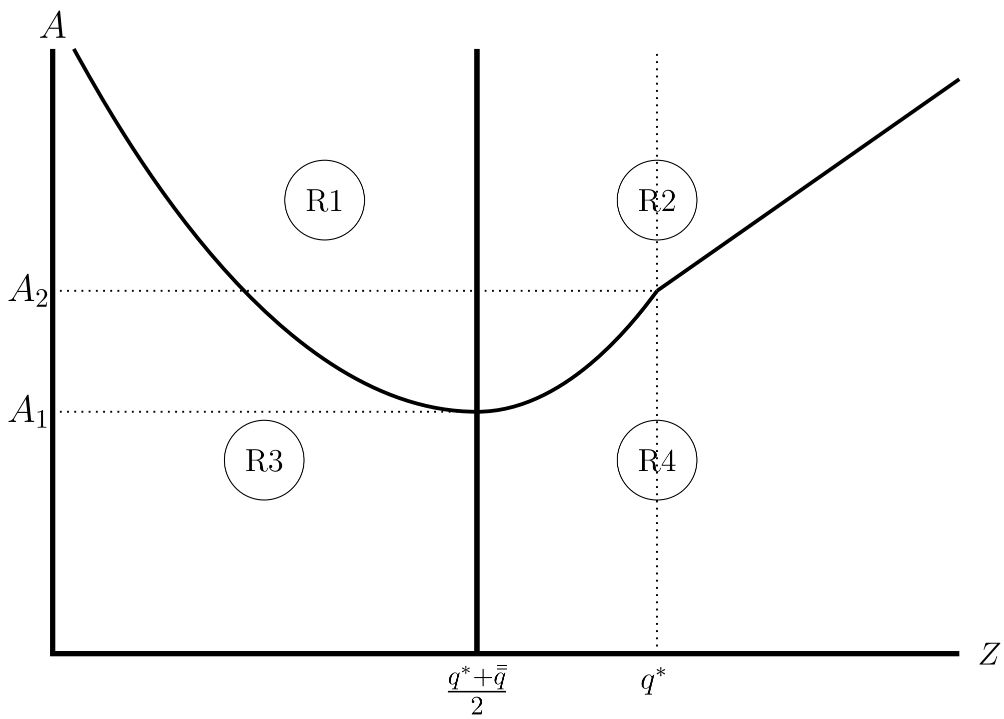


Figure 3: Aggregate regions of equilibrium (if $\rho = 1$)

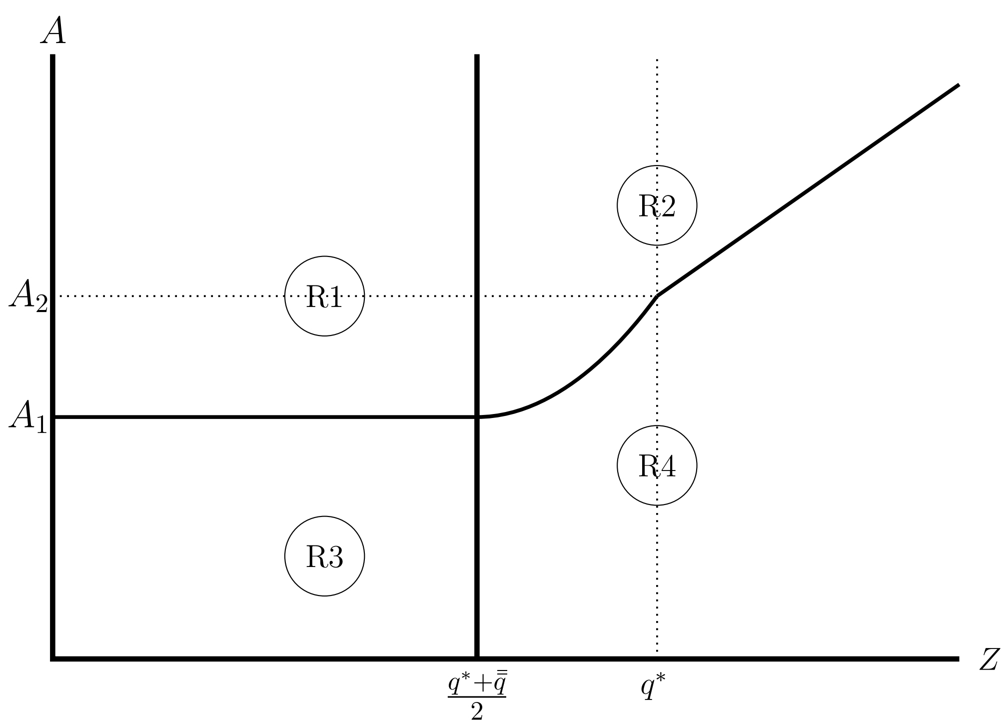
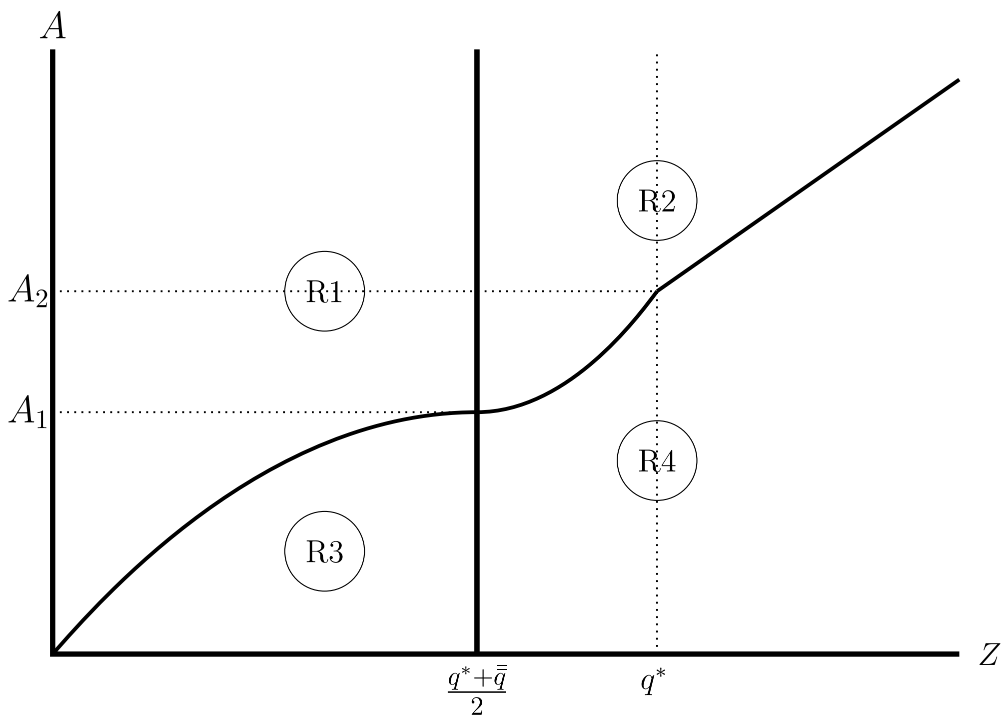


Figure 4: Aggregate regions of equilibrium (if $\rho < 1$)



a higher inflation pushes the equilibrium region into the one where total liquidity becomes scarce. For instance, when the nominal interest rate almost comes down to the Friedman Rule, the H-type can afford to buy up the entire asset A, i.e., Region 4. However, as inflation gets higher the H-type cannot purchase the whole A due to less holdings of money, but the L-type's post-OTC money holdings could still be the first best amount, i.e., Region 2. If inflation becomes so severe, then even the L-type cannot achieve the first best amount of liquidity after OTC trade, i.e., Region 1.

Proposition 2 explains the equilibrium behavior of OTC asset prices and trade volume, especially with regard to changes in inflation.

Figure 5: Aggregate regions of equilibrium in terms of μ (if $\rho > 1$)

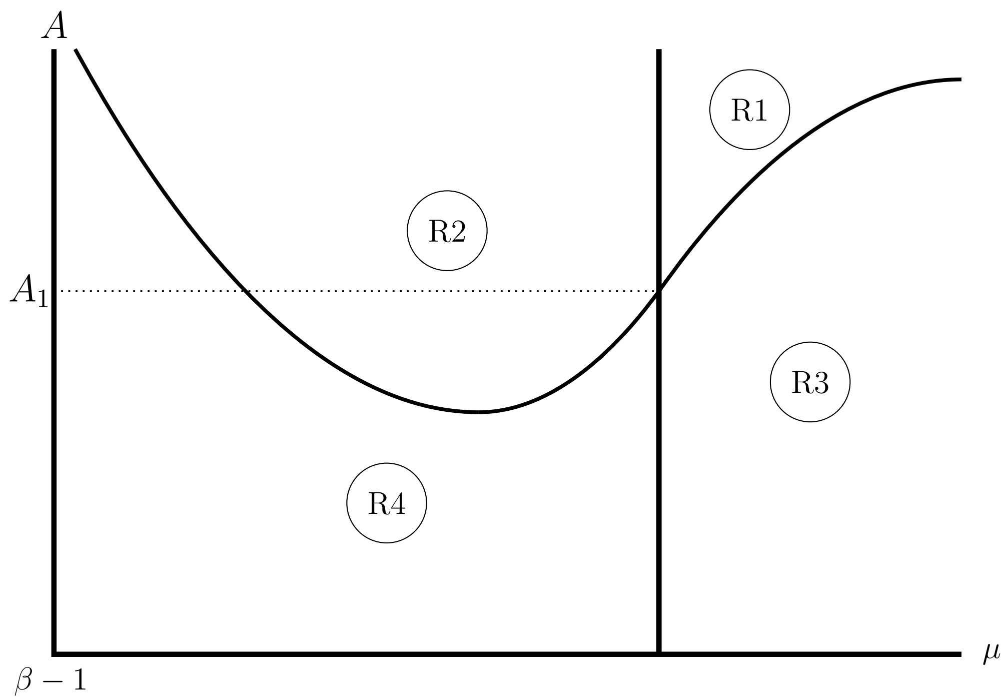
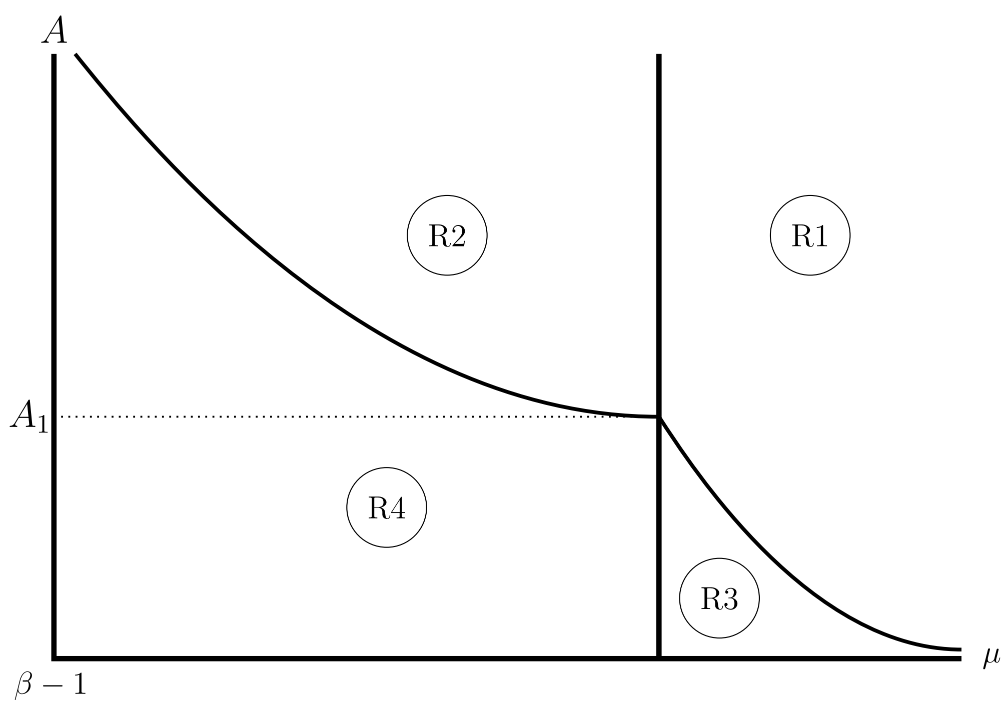


Figure 6: Aggregate regions of equilibrium in terms of μ (if $\rho \leq 1$)



Proposition 2 *The steady state OTC asset price (ψ) and trade volume (χ) exhibit the following properties:*
In Region 1, $\partial\psi/\partial\mu < 0$ and

$$\frac{\partial\chi}{\partial\mu} = \begin{cases} < 0, & \text{if } \rho < 1, \\ = 0, & \text{if } \rho = 1, \\ > 0, & \text{if } \rho > 1, \end{cases} \quad \frac{\partial^2\chi}{\partial\mu^2} = \begin{cases} > 0, & \text{if } \rho < 1, \\ = 0, & \text{if } \rho = 1, \\ < 0, & \text{if } \rho > 1, \end{cases}$$

In Region 2, $\partial\psi/\partial\mu \lesseqgtr 0$, and

$$\frac{\partial\chi}{\partial\mu} = \begin{cases} < 0, & \text{if } \mu \in (\mu_0, \mu_2] \text{ and } \rho \leq 1, \\ \lesseqgtr 0, & \text{if } \mu \in (\mu_0, \mu_2] \text{ and } \rho > 1. \end{cases}$$

In Region 3, $\partial\psi/\partial\mu < 0$ and $\partial\chi/\partial\mu = 0$. In Region 4, $\partial\psi/\partial\mu = 0$, and $\partial\chi/\partial\mu = 0$. Note that μ_0 is such that the steady state version of equation below is satisfied with $Z = \alpha_L A + \lambda^{-1}$ and μ_2 is such that the steady state version of equation below is satisfied with $Z = (1 + \lambda^{-1})/2$.

$$\frac{\varphi}{\beta\hat{\varphi}} = 1 + f\Big\{\frac{\alpha_H - \alpha_L}{\alpha_L}\Big\} + (1-f)\Big\{u'(q(\hat{m}))\frac{\partial q}{\partial \hat{m}}\frac{1}{\hat{\varphi}} - \frac{\partial p(\hat{m})}{\partial \hat{m}}\Big\}.$$

.

Proof. See Appendix A.3. ■

Equilibrium trade volume is non-varying in Region 3 and 4 where the H-type will be able to purchase the entire A regardless of inflation. In Region 1 and 2 where only a partial asset transfer occurs in OTC, a higher inflation can stimulate more asset transfer at the steady state only if $\rho > 1$. The intuitive explanation has already been suggested earlier. There are two offsetting inflation effects on OTC trade volume. First, a higher inflation makes the L-type become more desperate for extra cash holdings in the LW, so she will be willing to transfer more assets, i.e., a positive effect. On the other hand, a higher inflation reduces the H-type's money holdings in OTC, and therefore tends to prevent the L-type from transferring assets, i.e., a negative effect. When the extent to which the L-type's desire for extra cash holdings in the LW market gets relatively higher, i.e., $\rho > 1$, the first effect dominates. Thus, inflation and OTC trade volume will co-move in equilibrium, if and only if $\rho > 1$. Note that the OTC asset price usually decreases in inflation except for Region 2. This is a straightforward implication of such non-negative relationship between OTC trade volume and inflation in equilibrium.

4.2 Welfare

The goal of this section is to study the effect of inflation on equilibrium welfare. We first derive the steady state welfare function, which is given by

$$\begin{aligned}\mathcal{W}(Z) = {} & (1-2f)\left[u(\min\{q^*, Z\}) - \min\{q^*, Z\}\right] \\ & + f\left[u(\min\{\bar{\bar{q}}, Z-\eta\}) - \min\{\bar{\bar{q}}, Z-\eta\}\right] + f\left[u(\min\{q^*, Z+\eta\}) - \min\{q^*, Z+\eta\}\right] \\ & + \ell\alpha_H A + (1-\ell)\alpha_L A + f\chi(Z)(\alpha_H - \alpha_L). \end{aligned} \tag{8}$$

The details of this derivation are relegated to Appendix A.2. The first and second line represent total DM utilities and the third line represents total CM utilities. Equilibrium welfare in this economy is determined through two channels. The first one is a typical real money balance channel, captured by total DM utilities. The second one is novel and based on OTC asset transfers, captured by the last term in the third line of equation (8). As can be seen in this equation, welfare certainly increases as assets change hands from those who value them less (L-types) to those who value them more (H-types).

We are now ready to study the effect of inflation on equilibrium welfare in a series of propositions.

Proposition 3 *If $\rho \leq 1$ then, $\partial\mathcal{W}/\partial\mu < 0$ for Region 1 and 2. $\partial\mathcal{W}/\partial\mu < 0$ for Region 3 and 4 under any value for $\rho > 0$.*

Proof. As inflation reduces Z, the only way $\mathcal{W}$ can possibly increase in inflation is through $\partial\chi/\partial\mu > 0$. But, $\partial\chi/\partial\mu < 0$ for Region 1 when $\rho \leq 1$ and $\partial\chi/\partial\mu = 0$ in Region 3 and 4 from Proposition 2. This completes the proof. ■

The logic of this result is straightforward. The second welfare channel, i.e., OTC asset transfer channel, works in the same way as the typical real money balance channel. A higher inflation will not only reduce DM production but also OTC trade volume when agents' desire for extra cash holdings in the LW market relatively weakens, i.e., $\rho \leq 1$. Furthermore, the OTC asset transfer channel does not even exist under equilibrium in Region 3 and 4, where the entire asset holdings of the L-type are transferred regardless of inflation. Thus, inflation always reduces welfare in these cases.

Proposition 4 *Let $\hat{q}$ be such that $\alpha_L A = u(\hat{q}+\eta) - u(\hat{q})$ where $\eta = \hat{q}(\lambda-1)/(1+\lambda)$. Let μ_1 be such that the steady state version of equation below is satisfied with $Z = G(f, \alpha_H, \alpha_L)^{-1/\rho}$,* *where* $G(f, \alpha_H, \alpha_L) \equiv \frac{1-2f-2f(\alpha_H-\alpha_L)/\alpha_L}{1-2f-f(\alpha_H-\alpha_L)/\alpha_L}$.

$$\frac{\varphi}{\beta\hat{\varphi}} = 1 + f\Big\{\frac{\alpha_H - \alpha_L}{\alpha_L}\Big\} + (1-f)\Big\{u'(q(\hat{m}))\frac{\partial q}{\partial \hat{m}}\frac{1}{\hat{\varphi}} - \frac{\partial p(\hat{m})}{\partial \hat{m}}\Big\}.$$

Also, let μ_3 *be such that the steady state version of equation below is satisfied with* $Z = \hat{q}$.

$$\begin{aligned}\frac{\varphi}{\beta\hat{\varphi}} =& 1 + f\big\{u'(\hat{\varphi}(\hat{m} - \delta(\hat{m}, \ddot{m}))) - 1\big\} + f\big\{u'(\hat{\varphi}\hat{m}) - 1\big\} \\ &+ (1-2f)\Big\{u'(q(\hat{m}))\frac{\partial q}{\partial \hat{m}}\frac{1}{\hat{\varphi}} - \frac{\partial p(\hat{m})}{\partial \hat{m}}\Big\},\end{aligned}$$

where $\delta(\hat{m}, \ddot{m})$ *is given by Lemma 2. Then, under the 3 sufficient conditions specified below the following is true.*

- $\lim_{\mu\to\mu_2}(\partial\mathcal{W}/\partial\mu) > 0$, $\lim_{\mu\to\mu_3}(\partial\mathcal{W}/\partial\mu) < 0$, *and* $\partial^2\mathcal{W}/\partial\mu^2 < 0$ *within Region 1, i.e.,* $\mathcal{W}$ *is hump-shaped in* μ *within Region 1.*
- $\partial\mathcal{W}/\partial\mu > 0$ *for* $\mu \geq \mu_1$ *in Region 2.*

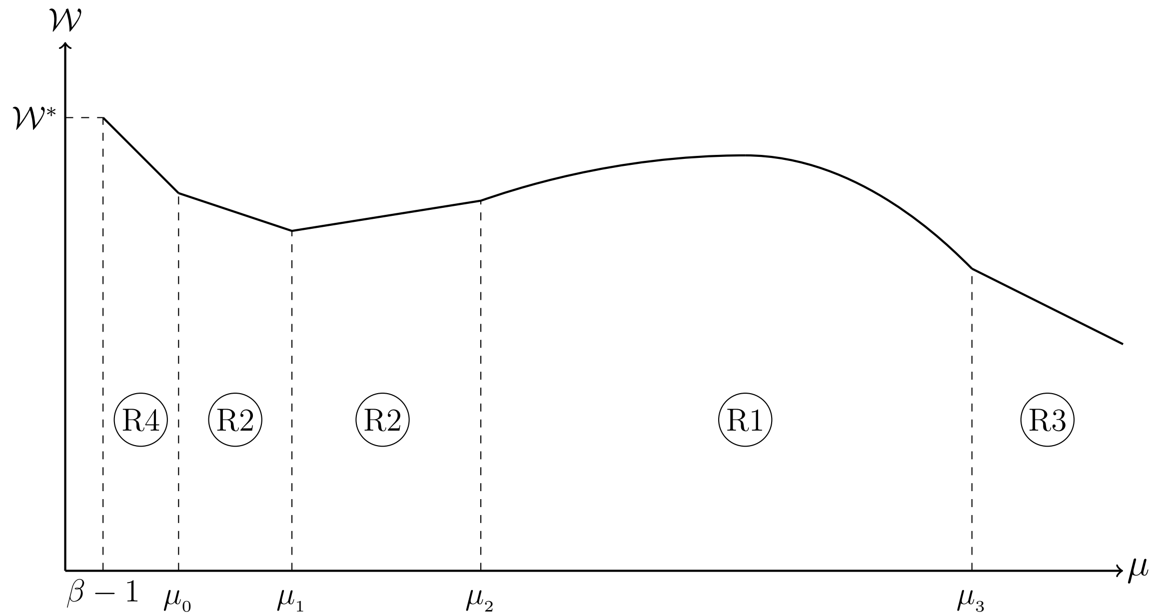
Sufficient Conditions:

1. $\rho > 1$
2. $f(\alpha_H - \alpha_L)/\alpha_L > 1 - 2f$
3. $\left(\frac{2\lambda}{1+\lambda}\right)^\rho > G(f, \alpha_H, \alpha_L)$

Proof. See Appendix A.3. ■

Intuitively, one needs relatively high enough risk aversion parameter ρ as well as asset valuation differential $\alpha_H - \alpha_L$ for Proposition 4 to hold true. Figure 7 can provide an intuitive explanation for Proposition 4. Starting from the Friedman rule, a higher inflation pushes equilibrium into Region 4 where all assets are traded in the OTC market. This kills the asset transfer effect on welfare. Thus, welfare is decreasing in inflation in this region. However, a further increase in inflation pushes equilibrium into Region 2, where only a partial asset transfer occurs. This is where the analysis becomes more interesting. When inflation is relatively low, in the precise sense that $\mu < \mu_1$, an increase in inflation induces the L-type to reduce the amount of transferred assets. This is because her pre-OTC real balances are still close to the first best, and therefore she will not be desperate for extra cash holdings in the LW market. Accordingly, the amount of asset transfer will only depend on the H-type's real balances. Hence, welfare decreases in inflation this region.

Figure 7: Steady state welfare as a function of μ under the 3 sufficient conditions



However, a further increase in inflation, i.e., one that brings us into the region where $\mu > \mu_1$, will increase the L-type's desire for extra cash holdings in the LW market, eventually inducing her to transfer more assets in the OTC market. Now, the positive welfare effect of OTC asset transfer starts dominating the real money balance effect, and therefore a higher inflation will lead to higher welfare in this region.[9] In fact, this dominance continues as we move into the interior of Region 1. Finally, transferable assets start to become depleted as equilibrium moves closer to Region 3. This kills off the positive welfare effect of asset transfer, and the relationship between inflation and welfare becomes negative again.

5 Numerical Analysis and Equilibrium Welfare

5.1 Calibration

Our objective in this section is to calibrate the model to U.S. data in order to provide a sense of the levels of inflation at which welfare is increasing in inflation. Following Lagos and Zhang (2020b) we set the period length equal to a day. This is consistent with high-frequency OTC trading activity we observe in practice. Time preference is described by the discount

[9] It should be highlighted that the timing of events in our model is crucial for this result, i.e., the OTC market must open before the DM. Inflation is beneficial because agents who find out they are L-types are inclined to sell assets in the OTC market and boost their money holdings *in the anticipation* of visiting the DM. If the DM opened before the OTC market, L-types would not have the opportunity to boost their liquidity by selling assets in the OTC, and this would eradicate the beneficial effects of higher inflation.

factor β. We assume the following functional form for the utility and the cost function in the LW market respectively: $u(x) = \varepsilon x^{1-\rho}/(1-\rho)$ and $c(q) = \varepsilon q$. The discount rate is determined so that $1/\beta$ equals the average daily real interest rate, adopted from Lagos and Zhang (2020b). The structural parameters that need to be calibrated are $\{\alpha_H, \alpha_L, \rho, \varepsilon, A\}$.

We set $\{\rho, \varepsilon\}$ in the following way. First, we assume the US economy is on average in equilibrium Region 1 so that the average real money balance $\bar{Z}$ is less than $(q^* + \bar{\bar{q}})/2$. Given the former, we find a combination of $\{\bar{Z}, \rho, \varepsilon\}$ that satisfies the following 2 conditions. First, in equilibrium Region 1, OTC volume to real balance ratio, i.e., η/Z, equals $(\lambda-1)/(\lambda+1)$, where $\lambda = (\alpha_H/\alpha_L)^{1/\rho}$ (See Proposition 2). So, we match $(\lambda-1)/(\lambda+1)$ with the ratio of daily US Treasury securities trading volume to M1, which is 0.1776 (in 2014). Second, we also match the steady state inflation in Region 1 from the model to the average daily U.S. inflation rate from Lagos and Zhang (2020b), $\bar{\mu} = 0.000073$. Then, the following condition, reflecting the steady-state version of first order condition with respect to money holdings, i.e., equation (a.2), should hold true.

$$\frac{1+\bar{\mu}}{\beta} = 1 + \frac{\varepsilon}{2}\left(\left[\frac{2\bar{Z}}{\lambda+1}\right]^{-\rho} - 1\right) + \frac{\varepsilon}{2}\left(\left[\frac{2\lambda\bar{Z}}{\lambda+1}\right]^{-\rho} - 1\right). \tag{9}$$

Next, we set A such that the average ratio between η and A, which equals $\bar{Z}(\lambda-1)/(A(\lambda+1))$ in the equilibrium Region 1 (See Proposition 2), matches with the ratio of average daily U.S. Treasury securities trading volume to U.S. Treasury securities outstanding, which is 0.0229 (in 2014).

Lastly, we set $\{\alpha_H, \alpha_L\}$ by satisfying the following two conditions. Based on the value of λ calibrated above, α_H can be expressed as $\lambda^\rho \alpha_L$ in the equilibrium Region 1. Second, following the standard monetary economics literature, we set α_L so that the average M/PY in the equilibrium Region 1, given by equation (10), matches the average $M1$ to GDP ratio, i.e., 0.16 (in 2014).

$$\frac{M}{PY} = \frac{\bar{Z}}{\frac{\lambda^\rho \alpha_L}{2}A + \frac{\alpha_L}{2}A + \frac{\alpha_H - \alpha_L}{2}\chi(\bar{Z}) + \bar{Z}}, \tag{10}$$

where

$$\chi(\bar{Z}) = \frac{\varepsilon \bar{Z}^{1-\rho}}{\alpha_L(1-\rho)}\left(\left(\frac{2\lambda}{1+\lambda}\right)^{1-\rho} - 1\right). \tag{11}$$

Note that equation (11) is a modified version of equation (a.10) in the proof for Proposition 2 since we here assume $c(q) = \varepsilon q$.

Table 1 summarizes the calibration targets and data sources. Calibrated parameters are summarized in Table 2.

Table 1: Calibration targets

Description	Value	Sources
Average daily real interest rate	0.00004	Lagos and Zhang (2020b)
Average daily net inflation rate	0.00007	Lagos and Zhang (2020b)
Average money demand $M1/PY$	0.16	FRED
$\frac{\text{US Treasury securities trading volume}}{\text{US Treasury securities outstanding}}$	0.9359	New York FED
$\frac{\text{US Treasury securities trading volume}}{\text{M1}}$	0.0229	New York FED & FRED

Table 2: Parameter values

Description		Value
Discount factor	β	0.9999
Preference parameter	ρ	5.5123
Scale parameter	ε	0.5
High valuation parameter	α_H	0.1645
Low valuation parameter	α_L	1.1904
Asset supply	A	3.9523

5.2 Welfare and Policy Implications

Based on the calibrated parameters we compute μ_2, i.e., the lower bound gross inflation rate for equilibrium Region 1 where $\mathcal{W}$ is hump-shaped in gross money growth rate. According to equation (a.4), μ_2 is such that

$$\frac{\mu_2}{\beta} = 1 + \frac{\varepsilon}{2}\left(\frac{\alpha_H - \alpha_L}{\alpha_L}\right) + \frac{\varepsilon}{2}\left(\bar{Z}_2^{-\rho} - 1\right), \tag{12}$$

where $\bar{Z}_2 = (q^* + \bar{\bar{q}})/2$. Based on equation (12), μ_2 turns out to be 0.99994, which implies an annualized net inflation rate of -2.2%. Figure 8 illustrates the steady state welfare as a function of daily gross inflation rate in equilibrium Region 1. It reveals that the range of inflation under which inflation is welfare improving is actually very large. The lower bound of this range is close to the Friedman rule, and the upper bound expands beyond a daily net inflation rate of 50%. (That is not to say that such levels of hyper-inflation are empirically relevant, but to say that the range for which equilibrium Region 1 applies is extremely large.) Therefore, our calibration exercise implies that our novel positive welfare effect of inflation is applicable at almost all plausible levels of inflation.

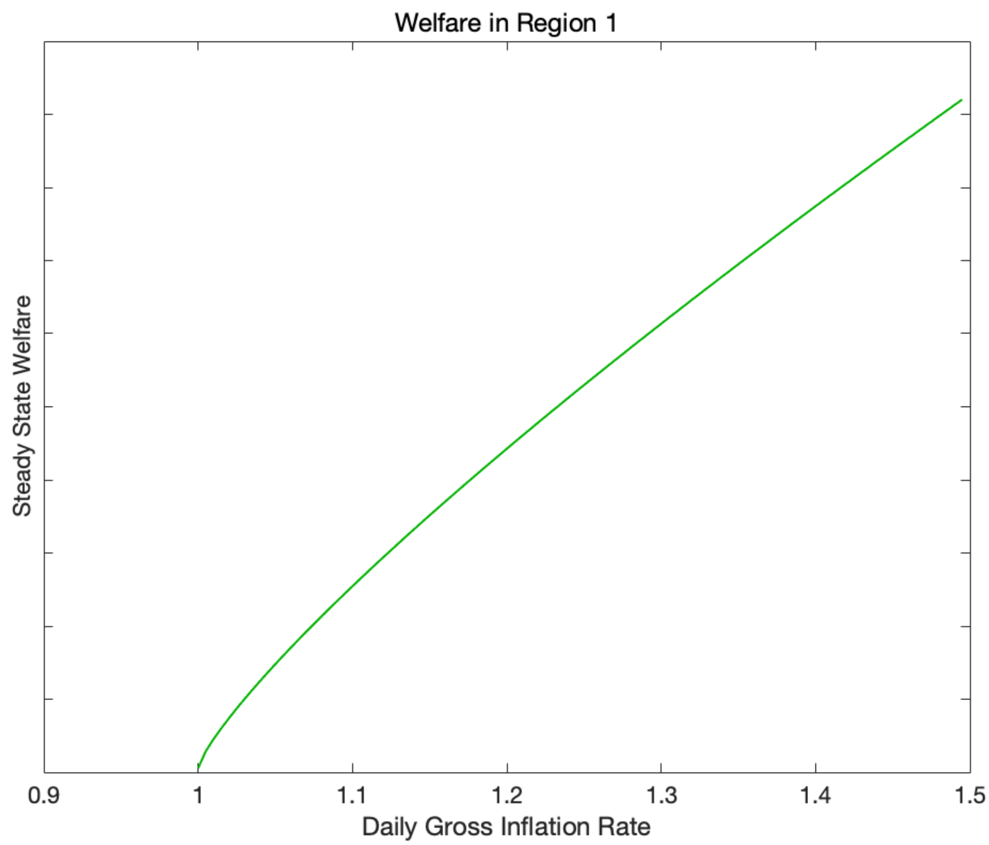


Figure 8: Steady-state welfare as a function of daily gross inflation rate in Region 1

6 Conclusion

An almost universal result in monetary theory is that higher inflation acts as a tax on real balances, thus, reducing equilibrium money holdings and trade, and hurting welfare. A common feature of the papers that reach this conclusion is that they adopt models where money serves as a medium of exchange exclusively in *goods markets*. Motivated by recent work in macroeconomics and finance, we model explicitly the idea that money serves as a medium of exchange in goods as well as asset markets. We highlight that this framework is not just more realistic, but also offers new economic insights regarding the effects of inflation on welfare and asset prices. The main result of the paper is that, contrary to the majority of the literature, in our model welfare can be increasing in inflation due to a new channel whereby higher inflation promotes beneficial trade in the secondary asset market. We describe in detail the parametric specifications under which this new and surprising result arises, and we also analyze the effects of inflation on asset prices and equilibrium asset trade volume.

References

Andolfatto, D., A. Berentsen, and C. Waller (2014): "Optimal disclosure policy and undue diligence," *Journal of Economic Theory*, 149, 128–152.

Berentsen, A., S. Huber, and A. Marchesiani (2014): "Degreasing the wheels of finance," *International economic review*, 55, 735–763.

Berentsen, A., G. Rocheteau, and S. Shi (2007): "Friedman meets Hosios: Efficiency in search models of money," *The Economic Journal*, 117, 174–195.

Chang, B. and S. Zhang (2015): "Endogenous market making and network formation," *Available at SSRN 2600242*.

Duffie, D., N. Gârleanu, and L. H. Pedersen (2005): "Over-the-Counter Markets," *Econometrica*, 73, 1815–1847.

Gabrovski, M. and I. Kospentaris (2021): "Intermediation in over-the-counter markets with price transparency," *Journal of Economic Theory*, 198, 105364.

Geromichalos, A. and L. Herrenbrueck (2016): "Monetary Policy, Asset Prices, and Liquidity in Over-the-Counter Markets," *Journal of Money, Credit and Banking*, 48, 35–79.

Geromichalos, A. and K. M. Jung (2018): "An Over-the-Counter Approach to the FOREX Market," *International Economic Review*, 59, 859–905.

——— (2019): "Monetary policy and efficiency in over-the-counter financial trade," *Canadian Journal of Economics/Revue canadienne d'économique*, 52, 1699–1754.

Geromichalos, A., K. M. Jung, S. Lee, and D. Carlos (2021): "A Model of Endogenous Direct and Indirect Asset Liquidity," *European Economic Review*, 132, 103627.

Geromichalos, A., J. M. Licari, and J. Suarez-Lledo (2007): "Monetary Policy and Asset Prices," *Review of Economic Dynamics*, 10, 761–779.

Han, H., B. Julien, A. Petursdottir, and L. Wang (2019): "Asset liquidity and indivisibility," *European Economic Review*, 119, 236–250.

Hu, T.-W. and G. Rocheteau (2014): "Monetary Policy and Asset Prices: A Mechanism Design Approach," *Journal of Money, Credit, and Banking, forthcoming.*

Jung, K. M. and J. H. Pyun (2016): "International Reserves for Emerging Economies: A Liquidity Approach," *Journal of International Money and Finance*, 68, 230–257.

Lagos, R. (2010): "Asset prices and liquidity in an exchange economy," *Journal of Monetary Economics*, 57, 913–930.

Lagos, R. and G. Rocheteau (2009): "Liquidity in asset markets with search frictions," *Econometrica*, 77, 403–426.

Lagos, R. and R. Wright (2005): "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113, 463–484.

Lagos, R. and S. Zhang (2019): "A monetary model of bilateral over-the-counter markets," *Review of Economic Dynamics*, 33, 205–227.

——— (2020a): "Turnover liquidity and the transmission of monetary policy," *American Economic Review*, 110, 1635–72.

——— (2020b): "Turnover Liquidity and the Transmission of Monetary Policy," *American Economic Review*, 110, 1635–1672.

Lee, S. and K. M. Jung (2020): "A Liquidity-Based Resolution of the Uncovered Interest Parity Puzzle," *Journal of Money, Credit and Banking*, 52, 1397–1433.

Lester, B., A. Postlewaite, and R. Wright (2012): "Information, liquidity, asset prices, and monetary policy," *The Review of Economic Studies*, 79, 1209–1238.

Madison, F. (2019): "Frictional asset reallocation under adverse selection," *Journal of Economic Dynamics and Control*, 100, 115–130.

Mattesini, F. and E. Nosal (2015): "Liquidity and asset prices in a monetary model with OTC asset markets," *Journal of Economic Theory*, doi:10.1016/j.jet.2015.11.001.

Molico, M. (2006): "The distribution of money and prices in search equilibrium," *International Economic Review*, 47, 701–722.

Nosal, E. and G. Rocheteau (2011): *Money, payments, and liquidity*, MIT press.

——— (2013): "Pairwise trade, asset prices, and monetary policy," *Journal of Economic Dynamics and Control*, 37, 1–17.

Rocheteau, G. (2011): "Payments and liquidity under adverse selection," *Journal of Monetary Economics*, 58, 191–205.

Rocheteau, G., P.-O. Weill, and T.-N. Wong (2015): "Long-run and short-run effects of money injections," Tech. rep., Working Paper.

Rocheteau, G. and R. Wright (2005): "Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium," *Econometrica*, 73, 175–202.

Üslü, S. (2019): "Pricing and liquidity in decentralized asset markets," *Econometrica*, 87, 2079–2140.

Weill, P.-O. (2007): "Leaning against the wind," *The Review of Economic Studies*, 74, 1329–1354.

Wright, R., S. X. Xiao, and Y. Zhu (2020): "Frictional capital reallocation with ex post heterogeneity," *Review of Economic Dynamics*, 37, S227–S253.

A Appendix

A.1 Optimal Behavior of Agents

We focus on prices that satisfy $\varphi > \beta\hat{\varphi}$, since we know that this will be always true in steady state equilibria with $\mu > \beta - 1$. Given the agent's belief $\ddot{m}$, she can end up in different branches of the bargaining solution, depending on her own choices $\hat{m}$ as well as asset endowment A. In general, the domain of the objective function can be divided into 4 regions. The following Lemma 3 shows the optimal money demand satisfies the following condition in each region.

Lemma 3 *Taking prices $(\varphi, \hat{\varphi})$, asset endowment A, and belief, $\ddot{m}$ as given, the optimal choice of the representative agent, $\hat{m}$, satisfies:*
R1: $\hat{m} + \ddot{m} < m^* + \bar{m}$ *and* $A \geq \bar{A}$ *where*

$$\bar{A} = \frac{u(\hat{\varphi}(\tilde{m} + \Lambda(m, \tilde{m}))) - u(q(\tilde{m}))}{\alpha_{L}}, \tag{a.1}$$

and m and $\tilde{m}$ denotes the H-type's and the L-type's money holdings in OTC respectively. $\Lambda(m, \tilde{m})$ is from Lemma 2. $\hat{m}$ should satisfy:

$$\begin{aligned}\frac{\varphi}{\beta\hat{\varphi}} =& 1 + f\Big\{u'(\hat{\varphi}(\hat{m} - \delta(\hat{m}, \ddot{m}))) - 1\Big\} + f\Big\{u'(\hat{\varphi}\hat{m}) - 1\Big\} \\ &+ (1 - 2f)\Big\{u'(q(\hat{m}))\frac{\partial q}{\partial \hat{m}}\frac{1}{\hat{\varphi}} - \frac{\partial p(\hat{m})}{\partial \hat{m}}\Big\},\end{aligned} \tag{a.2}$$

where $\delta(\hat{m}, \ddot{m})$ is given by Lemma 2.
R2: $\hat{m} + \ddot{m} \geq m^* + \bar{m}$ *and* $A \geq \bar{A}$ *where*

$$\bar{A} = \frac{u(q^*) - q^* - u(q(\tilde{m})) + \hat{\varphi}p(\tilde{m}) + \hat{\varphi}\Lambda(m, \tilde{m})}{\alpha_{L}}, \tag{a.3}$$

and m and $\tilde{m}$ denotes the H-type's and the L-type's money holdings in OTC respectively. $\Lambda(m, \tilde{m})$ is from Lemma 2. $\hat{m}$ should satisfy:

$$\frac{\varphi}{\beta\hat{\varphi}} = 1 + f\Big\{\frac{\alpha_{H} - \alpha_{L}}{\alpha_{L}}\Big\} + (1 - f)\Big\{u'(q(\hat{m}))\frac{\partial q}{\partial \hat{m}}\frac{1}{\hat{\varphi}} - \frac{\partial p(\hat{m})}{\partial \hat{m}}\Big\}. \tag{a.4}$$

R3: $\hat{m}+\ddot{m}<m^*+\bar{m}$ *and* $A<\bar{A}$ *where* $\bar{A}$ *is given in the eq.(a.1).* $\hat{m}$ *should satisfy:*

$$\begin{aligned}\frac{\varphi}{\beta\hat{\varphi}} =&1+f\Big\{u'(\hat{\varphi}(\hat{m}-\delta(\hat{m},\ddot{m})))-1\Big\}+f\Big\{u'(\hat{\varphi}\hat{m})-1\Big\}\\ &+(1-2f)\Big\{u'(q(\hat{m}))\frac{\partial q}{\partial \hat{m}}\frac{1}{\hat{\varphi}}-\frac{\partial p(\hat{m})}{\partial \hat{m}}\Big\},\end{aligned} \tag{a.5}$$

where δ *are from Lemma 2.*

R4: $\hat{m}+\ddot{m}\geq m^*+\bar{m}$ *and* $A<\bar{A}$ *where* $\bar{A}$ *is given in the eq.(a.3).* $\hat{m}$ *should satisfy:*

$$\frac{\varphi}{\beta\hat{\varphi}} =1+f\Big\{u'(\hat{\varphi}(\hat{m}-\delta(\hat{m},\ddot{m})))-1\Big\}+(1-f)\Big\{u'(q(\hat{m}))\frac{\partial q}{\partial \hat{m}}\frac{1}{\hat{\varphi}}-\frac{\partial p(\hat{m})}{\partial \hat{m}}\Big\}. \tag{a.6}$$

where δ *are from Lemma 2.*

A.2 Derivation of the $\mathcal{W}$ function in Section 5.2

First, we let X^k_{Ci} (H^k_{Ci}), $i\in\{H,L\}$ and $k\in\{m,n\}$ denote the equilibrium CM consumption (work effort) for the agent type i whose previous OTC trading status was k (where m stands for a successful OTC match, while n does for an unsuccessful OTC match). Likewise, we let X^k_{Pj} (H^k_{Pj}), $j\in\{H,L\}$ and $k\in\{m,n\}$ denote the equilibrium CM consumption (work effort) of the producer who matched with a type j buyer, whose OTC trading status is k.

Let $\mathcal{C}_C$ denote the total net CM utilities of (all) agents. Then, we obtain

$$\begin{aligned}\mathcal{C}_C =&\ell(1-\pi_H)(X^n_{CH}-H^n_{CH})+\ell\pi_H(X^m_{CH}-H^m_{CH})\\ &+(1-\ell)\pi_L(X^m_{CL}-H^m_{CL})+(1-\ell)(1-\pi_L)(X^n_{CL}-H^n_{CL})\\ &+\ell(1-\pi_H)X^n_{PH}+\ell\pi_H X^m_{PH}+(1-\ell)(1-\pi_L)X^n_{PL}+(1-\ell)\pi_L X^m_{PL}.\end{aligned}$$

Note that producer's work effort (H^k_{Pj}) always equal zero at the steady state equilibrium. The followings are the breakdown of X^k_{Ci} and H^k_{Ci} for different agents with different trading histories.

Matched type-H agents and producers

$$X^m_{CH}=\alpha_H[A+\chi(Z)],\ \ H^m_{CH}=Z,\ \text{ and }\ X^m_{PH}=(Z-\eta).$$

Matched type-L agents and producers

$$X^m_{CL}=\alpha_L[A-\chi(Z)],\ \ H^m_{CL}=Z,\ \text{ and }\ X^m_{PL}=(Z+\eta).$$

Unmatched type-H agents and producers

$$X^n_{BH} = \alpha_H A, \;\; H^n_{CH} = Z, \;\; \text{and} \;\; X^n_{PH} = Z.$$

Unmatched type-L agents and producers

$$X^n_{CL} = \alpha_L A, \;\; H^n_{CL} = Z, \;\; \text{and} \;\; X^n_{PL} = Z.$$

Combining all these leads to $\mathcal{C}_C = \ell\alpha_H A + (1-\ell)\alpha_L A + f\chi(Z)(\alpha_H - \alpha_L)$. Net DM utilities for each type of agents follow as below.

Matched type-H agents

$$U^m_{CH} = \ell\pi_H \left(u(\min\{\bar{\bar{q}}, Z-\eta\}) - \min\{\bar{\bar{q}}, Z-\eta\}\right).$$

Matched type-L agents

$$U^m_{CL} = (1-\ell)\pi_L \left(u(\min\{q^*, Z+\eta\}) - \min\{q^*, Z+\eta\}\right).$$

Unmatched type-H agents

$$U^n_{CH} = \ell(1-\pi_H)\left(u(\min\{q^*, Z\}) - \min\{q^*, Z\}\right).$$

Unmatched type-L agents and producers

$$U^n_{CL} = (1-\ell)(1-\pi_L)\left(u(\min\{q^*, Z\}) - \min\{q^*, Z\}\right).$$

If one adds total net DM utilities to $\mathcal{C}_C$, one finally gets eq.(8).

A.3 Proofs of Statements

Proof of Lemma 2.

In the bargaining game between a seller, i.e., the L-type, and a buyer, i.e., the H-type, the Lagrangian function, ignoring the money constraint, becomes

$$\begin{aligned}\mathcal{L} = & \, u\left(q\left(m-\delta\right)\right) - u\left(q(m)\right) - \varphi\delta + \varphi\left[p(m) - p\left(m-\delta\right)\right] \\ & + \frac{\alpha_H}{\alpha_L}\Big\{u\left(q(\tilde{m}+\delta)\right) - u\left(q(\tilde{m})\right) + \varphi\delta - \varphi\left[p(\tilde{m}+\delta) - p(\tilde{m})\right]\Big\} \\ & + \tau\Big\{A - \Big(\varphi\delta + u\left(q(\tilde{m}+\delta)\right) - u\left(q(\tilde{m})\right) - \varphi[p(\tilde{m}+\delta) - p(\tilde{m})]\Big)/\alpha_L\Big\},\end{aligned}$$

where τ denotes the Lagrangian multiplier on the resource constraint, i.e., $A \geq \chi$, and χ is equivalent to the one implied by the L-type's trading surplus being 0. The corresponding FOC with respect to δ is given by

$$\begin{aligned}\delta : 0 = &- u'\left(q(m-\delta)\right)\right) \frac{\partial q}{\partial(m-\delta)} + \varphi \frac{\partial p}{\partial(m-\delta)} - \varphi \\ &+ \frac{\alpha_H}{\alpha_L}\left[u'\left(q(\tilde{m}+\delta)\right)\right) \frac{\partial q}{\partial(\tilde{m}+\delta)} - \varphi \frac{\partial p}{\partial(\tilde{m}+\delta)} + \varphi\right] \\ &- \frac{\tau}{\alpha_L}\left[u'\left(q(\tilde{m}+\delta)\right) \frac{\partial q}{\partial(\tilde{m}+\delta)} - \varphi \frac{\partial p}{\partial(\tilde{m}+\delta)} + \varphi\right].\end{aligned} \tag{a.7}$$

We then analyze each case separately.

Case 1: $\tau = 0 \Rightarrow A > \chi$.

Sub-case 1.1: $\tilde{m} + \delta \geq m^*$ and $m - \delta \geq m^*$

In this case, the eq.(a.7) gives $0 = -\varphi + \frac{\alpha_H}{\alpha_L}\varphi$, which is not possible. So, this sub-case is not feasible.

Sub-case 1.2: $\tilde{m} + \delta < m^*$ and $m - \delta \geq m^*$

The eq.(a.7) gives $\frac{\alpha_H}{\alpha_L} u'(\varphi(\tilde{m} + \delta)) = 1$, which is not possible. So, this sub-case is not feasible.

Sub-case 1.3: $m - \delta < m^*$ and $\tilde{m} + \delta \geq m^*$

The eq.(a.7) gives

$$u'\left(\varphi(m-\delta)\right) = \frac{\alpha_H}{\alpha_L}, \tag{a.8}$$

which also requires $m \geq \bar{m}$, otherwise δ will not exist. Note that $\delta = m - \bar{m}$. Since $\tilde{m} + \delta \geq m^*$, $m + \tilde{m} \geq m + \bar{m}$. Finally, combining eq.(a.8) with the L-type's participation constraint, one should get $\alpha_L \chi = u(q^*) - q^* - u(q(\tilde{m})) + \varphi\left(p(\tilde{m}) + \Lambda\right)$, where Λ equals δ in eq.(a.8).

Sub-case 1.4: $m - \delta < m^*$ and $\tilde{m} + \delta < m^*$

The eq.(a.7) gives

$$\frac{u'\left(\varphi(m-\delta)\right)}{u'\left(\varphi(\tilde{m}+\delta)\right)} = \frac{\alpha_H}{\alpha_L}, \tag{a.9}$$

which also requires $m + \tilde{m} < m^* + \bar{m}$ due to similar reason in the sub-case 1.3 earlier. Finally, eq.(a.9) implies $\alpha_L \chi = u(\varphi(\tilde{m} + \delta)) - u(\varphi\tilde{m})$, where δ is from eq.(a.9).

Case 2: $\tau > 0 \Rightarrow A = \chi$.

Sub-case 2.1: $m - \delta \geq m^*$ and $\tilde{m} + \delta \geq m^*$

Here, $m + \tilde{m} \geq 2m^*$. The eq.(a.7) gives $\tau = \alpha_H - \alpha_L$. Then, due to the L-type's participation constraint δ is such that $\alpha_L A = u(q^*) - q^* - u(q(\tilde{m})) + \varphi p(\tilde{m}) + \varphi\delta$.

Sub-case 2.2: $\bar{m} \leq m - \delta < m^*$ and $\tilde{m} + \delta \geq m^*$

This case is consistent with $m + \tilde{m} \geq m^* + \bar{m}$. Equation (a.7) gives $0 = u'(\varphi(m - \delta))\varphi + ((\alpha_H - \tau)/\alpha_L)\varphi$, which combined with the L-type's participation constraint implies $\alpha_L A = u(q^*) - q^* - u(q(\tilde{m})) + \varphi p(\tilde{m}) + \varphi\delta$.

Sub-case 2.3: $m - \delta < \bar{m}$ and $\tilde{m} + \delta \geq m^*$

Equation (a.7) gives $0 = u'(\varphi(m-\delta))\varphi + ((\alpha_H - \tau)/\alpha_L)\varphi$, which is a contradiction since $u'(\varphi(m - \delta)) > u'(\bar{m}) = \alpha_H/\alpha_L > (\alpha_H - \tau)/\alpha_L$.

Sub-case 2.4: $m - \delta \geq m^*$ and $\tilde{m} + \delta < m^*$

The eq.(a.7) gives $((\alpha_H - \tau)/\alpha_L)\, u'(\varphi(\tilde{m} + \delta)) = 1$. Note that this sub-case implies $m + \tilde{m} < m^* + \bar{m}$ since $\tilde{m} + \delta < \bar{m}$ due to the FOC above. Hence, combining with the L-type's participation constraint, δ should solve $\alpha_L A = u(\varphi(\tilde{m}+\delta)) - u(\varphi\tilde{m})$. Since $\Lambda = \max\{\lambda\}$ here, the following should hold. $\alpha_L A < u(\varphi(\tilde{m} + \Lambda)) - u(\varphi\tilde{m})$, where Λ satisfies $u'(q(m - \Lambda)) = (\alpha_H/\alpha_L)u'(q(\tilde{m} + \Lambda))$.

Sub-case 2.5: $m - \delta < m^*$ and $\tilde{m} + \delta < m^*$

The eq.(a.7) gives $((\alpha_H - \tau)/\alpha_L)\, u'(\varphi(\tilde{m} + \delta)) = u'(\varphi(m - \delta)) < \alpha_H/\alpha_L$, which implies $\tilde{m} + \delta < \bar{m}$ and in turn $m + \tilde{m} < m^* + \bar{m}$. Hence, combining with the L-type's participation constraint, δ should solve $\alpha_L A = u(\varphi(\tilde{m} + \delta)) - u(\varphi\tilde{m})$. Since $\Lambda = \max\{\delta\}$ here, the following should hold. $\alpha_L A < u(\varphi(\tilde{m} + \Lambda)) - u(\varphi\tilde{m})$, where Λ satisfies $u'(q(m - \Lambda)) = (\alpha_H/\alpha_L)u'(q(\tilde{m} + \Lambda))$. ■

Proof of Lemma 3.

Region 1: In this region, post-OTC money holdings for both types fall short of m^*. If the agent happens to be the L-type, then $\tilde{\delta}$ is such that $u'(\hat{\varphi}(\ddot{m} - \tilde{\delta}))/u'(\hat{\varphi}(\hat{m} + \tilde{\delta})) = \alpha_H/\alpha_L$, and $\tilde{\chi}$ is such that $\alpha_L\tilde{\chi} = u(\hat{\varphi}(\hat{m} + \tilde{\delta})) - u(\hat{\varphi}\hat{m})$. If the agent happens to be the H-type, then δ is such that $u'(\hat{\varphi}(\hat{m} - \delta))/u'(\hat{\varphi}(\ddot{m} + \delta)) = \alpha_H/\alpha_L$, and χ is such that $\alpha_L\chi = u(\hat{\varphi}(\ddot{m} + \delta)) - u(\hat{\varphi}\ddot{m})$, where the following derivatives can be obtained.

$$\frac{\partial\delta(\hat{m}, \ddot{m})}{\partial\hat{m}} = \frac{u''(\hat{\varphi}(\hat{m} - \delta(\hat{m}, \ddot{m})))}{\frac{\alpha_H}{\alpha_L}u''(\hat{\varphi}(\ddot{m} + \delta(\hat{m}, \ddot{m})) + u''(\hat{\varphi}(\hat{m} - \delta(\hat{m}, \ddot{m}))} > 0,$$

$$\frac{\partial\tilde{\delta}(\ddot{m}, \hat{m})}{\partial\hat{m}} = \frac{-\frac{\alpha_H}{\alpha_L}u''(\hat{\varphi}(\hat{m} + \tilde{\delta}(\ddot{m}, \hat{m})))}{\frac{\alpha_H}{\alpha_L}u''(\hat{\varphi}(\hat{m} + \tilde{\delta}(\ddot{m}, \hat{m})) + u''(\hat{\varphi}(\ddot{m} - \tilde{\delta}(\ddot{m}, \hat{m}))} > 0,$$

$$\frac{\partial\chi}{\partial\hat{m}} = \frac{u'(\hat{\varphi}(\ddot{m} + \delta(\hat{m}, \ddot{m})))\frac{\partial\delta(\hat{m}, \ddot{m})}{\partial\hat{m}}\hat{\varphi}}{\alpha_L} > 0,$$

$$\frac{\partial\tilde{\chi}}{\partial\hat{m}} = u'(\hat{\varphi}(\hat{m} + \delta(\ddot{m}, \hat{m})))\hat{\varphi}\left(1 + \frac{\partial\delta(\ddot{m}, \hat{m})}{\partial\hat{m}}\right) - u'(q(\hat{m})) < 0.$$

Using these derivatives, the following conditions can be derived.

$$u'(\hat{\varphi}(\hat{m}-\delta))\hat{\varphi}(1-\frac{\partial\delta}{\partial\hat{m}})+\alpha_{H}\frac{\partial\chi}{\partial\hat{m}}=\hat{\varphi}u'(\hat{\varphi}(\hat{m}-\delta)),$$
$$u'(\hat{\varphi}(\hat{m}+\tilde{\delta}))\hat{\varphi}(1+\frac{\partial\tilde{\delta}}{\partial\hat{m}})-\alpha_{L}\frac{\partial\tilde{\chi}}{\partial\hat{m}}=\hat{\varphi}u'(\hat{\varphi}\hat{m}).$$

Combining these results with the first derivative of eq.(7) with respect to $\hat{m}$ gives the optimal condition eq.(a.2). The uniqueness of $\hat{m}$ and a negative relationship with $\varphi/(\beta\hat{\varphi})$ are easily established from the concavity of u.

Region 2: In this region, post-OTC money holdings for the H-type fall short of m^*, whereas post-OTC holdings for the L-type become greater than or equal to m^*. If the agent happens to be the L-type, then $\tilde{\delta}$ is such that $u'(\hat{\varphi}(\ddot{m}-\tilde{\delta}))=\alpha_{H}/\alpha_{L}$, and $\tilde{\chi}$ is such that $\alpha_{L}\tilde{\chi}=u(q(\ddot{m}))-u(q(\ddot{m}-\tilde{\delta}))$. If the agent happens to be the H-type, then δ is such that $u'(\hat{\varphi}(\hat{m}-\delta))=\alpha_{H}/\alpha_{L}$, and χ is such that $\alpha_{L}\chi=\hat{\varphi}\delta$, where $\partial\tilde{\delta}/\partial\hat{m}=0$, $\partial\delta/\partial\hat{m}=1$, and $\partial\chi/\partial\hat{m}=\hat{\varphi}/\alpha_{L}$, $\partial\tilde{\chi}/\partial\hat{m}=-(u'(\hat{\varphi}\hat{m})\hat{\varphi}-\hat{\varphi})/\alpha_{L}$. Combining these results with the first derivative of eq.(7) with respect to $\hat{m}$ gives the optimal condition eq.(a.4). The uniqueness of $\hat{m}$ and a negative relationship between $\varphi/(\beta\hat{\varphi})$ are easily established from the concavity of u.

Region 3: In this region, post-OTC money holdings for both types fall short of m^*. If the agent happens to be the L-type, then $\tilde{\chi}=A$ and $\tilde{\delta}$ is such that $\alpha_{L}A=u(\hat{\varphi}(\hat{m}+\tilde{\delta}))-u(\hat{\varphi}\hat{m})$. If the agent happens to be the H-type, then $\chi=A$. δ is such that $\alpha_{L}A=u(\hat{\varphi}(\ddot{m}+\delta))-u(\hat{\varphi}\ddot{m})$. Combining these results with the first derivative of eq.(7) with respect to $\hat{m}$ gives eq.(a.5). The uniqueness of $\hat{m}$ and a negative relationship between $\varphi/(\beta\hat{\varphi})$ are easily established from the concavity of u.

Region 4: In this region, post-OTC money holdings for the H-type fall short of m^*, whereas post-OTC holdings for the L-type become greater than or equal to m^*. If the agent happens to be the L-type, then $\tilde{\chi}=A$ and $\tilde{\delta}$ is such that $\alpha_{L}A=u(q^*)-q^*-u(q(\hat{m}))+\hat{\varphi}p(\hat{m})+\hat{\varphi}\tilde{\delta}$. If the agent happens to be the H-type, then $\chi=A$. δ is such that $\alpha_{L}A=u(q^*)-q^*-u(q(\ddot{m}))+\hat{\varphi}p(\ddot{m})+\hat{\varphi}\delta$. Combining these results with the first derivative of eq.(7) with respect to $\hat{m}$ gives eq.(a.6). The uniqueness of $\hat{m}$ and a negative relationship between $\varphi/(\beta\hat{\varphi})$ are easily established from the concavity of u. ■

Proof of Proposition 1.

Proof for part a) follows from the steady state version of OTC bargaining solutions from Lemma 2. η equals $\varphi\delta$ in the steady state equilibrium. Then, in the equilibrium Region 1

$$\frac{\partial\chi}{\partial Z}=\frac{u'(Z+\eta)(1+\frac{\partial\eta}{\partial Z})-u'(Z)}{\alpha_{L}},$$

Also, $u'(Z-\eta)/u'(Z+\eta) = \alpha_{_H}/\alpha_{_L}$. Then, $\partial\eta/\partial Z = (-x+y)/(x+y)$, where $x = \alpha_{_H} u''(Z+\eta) < 0$ and $y = \alpha_{_L} u''(Z-\eta) < 0$. So, if $|y| > |x|$, $\partial\eta/\partial Z > 0$. If the u function exhibits CRRA then

$$\frac{Z+\eta}{Z-\eta}\frac{u''(Z+\eta)}{u''(Z-\eta)}\frac{u'(Z-\eta)}{u'(Z+\eta)} = \rho.$$

Thus, $\alpha_{_H} u''(Z+\eta)/(\alpha_{_L} u''(Z-\eta)) < 1$,which implies $|y| > |x|$. Therefore, $\partial\eta/\partial Z > 0$. Using the definition of relative risk aversion, it can be also shown that $\partial\eta/\partial Z = \eta/Z$. Hence, the following should hold true as well. $\partial\chi/\partial Z = u'(Z+\eta)((Z+\eta)/Z) - u'(Z)/\alpha_{_L}$. This leads to the following.

$$\frac{\partial\chi}{\partial Z} = \begin{cases} > 0, & \text{if } \ q_1 u'(q_1) > q_2 u'(q_2), \\ = 0, & \text{if } \ q_1 u'(q_1) = q_2 u'(q_2), \\ < 0, & \text{if } \ q_1 u'(q_1) < q_2 u'(q_2), \end{cases}$$

where $q_1 > q_2$. Also,

$$\begin{cases} q_1 u'(q_1) > q_2 u'(q_2), & \text{if } \ \rho < 1, \\ q_1 u'(q_1) = q_2 u'(q_2), & \text{if } \ \rho = 1, \\ q_1 u'(q_1) < q_2 u'(q_2), & \text{if } \ \rho > 1. \end{cases}$$

Lastly, we prove why

$$\frac{\partial^2\chi}{\partial Z^2} = \begin{cases} > 0, & \text{if } \rho > 1, \\ = 0, & \text{if } \rho = 1, \\ < 0, & \text{if } \rho < 1, \end{cases}$$

Assuming $u(q) = q^{1-\rho}/(1-\rho)$, $\eta = ((1-\mu)/(1+\mu))Z$, where $\mu = (\alpha_{_H}/\alpha_{_L})^{-1/\rho} < 1$, $\partial^2\eta/\partial Z^2 = 0$. Also, this leads to

$$\frac{\partial^2\chi}{\partial Z^2} = \frac{u''(Z+\eta)((Z+\eta)/Z)^2 - u''(Z)\}}{\alpha_{_L}}.$$

Let $q_1 > q_2 > 0$. Then,

$$\begin{cases} q_1^2 u''(q_1) > q_2^2 u''(q_2), & \text{if } \ \rho > 1, \\ q_1^2 u''(q_1) = q_2^2 u''(q_2), & \text{if } \ \rho = 1, \\ q_1^2 u''(q_1) < q_2^2 u''(q_2), & \text{if } \ \rho < 1. \end{cases}$$

This completes the proof for part b). For part c) the following condition is needed. $(Z - \eta)^{-\rho} = \alpha_H/\alpha_L$, which leads to $\eta = Z - \lambda^{-1}$ where $\lambda = (\alpha_H/\alpha_L)^{1/\rho} > 1$. Thus, χ here is given by

$$\chi = \frac{1}{\alpha_L}\Big\{u(q^*) - q^* - \frac{Z^{1-\rho}}{1-\rho} + 2Z - \lambda^{-1}\Big\}.$$

Taking derivatives of χ with respect to Z proves part c). ■

Proof of Proposition 2.
In Region 1, $\eta = ((\lambda-1)/(1+\lambda))Z$ and

$$\chi = \frac{Z^{1-\rho}}{\alpha_L(1-\rho)}\left(\left(\frac{2\lambda}{1+\lambda}\right)^{1-\rho} - 1\right). \tag{a.10}$$

This proves why $\partial\psi/\partial\mu < 0$. Proof for $\partial\chi/\partial\mu$ easily follows from Proposition 1. In Region 2, $\eta = Z - \lambda^{-1}$, $\alpha_L\chi = \rho/(1-\rho) - Z^{1-\rho}/(1-\rho) + 2Z - \lambda^{-1}$, which provides a proof for $\partial\psi/\partial\mu$. Proof for $\partial\chi/\partial\mu$ easily follows from Proposition 1. In Region 3, $\chi = A$ and

$$\frac{\partial\eta}{\partial Z} = -\frac{(Z+\eta)^{\rho} - Z^{\rho}}{(Z+\eta)^{-\rho}} > 0.$$

Thus, $\partial\psi/\partial\mu < 0$. In Region 4, $\chi = A$ and $\eta = \alpha_L A$. Thus, $\partial\psi/\partial\mu = 0$. $\mu_0(\mu_2)$ is the cutoff level of money growth rate that divides Region 4 and 2 (Region 2 and 1) in Figure 5 respectively. ■

Proof of Proposition 4. Under the equilibrium Region 1, eq.(8) leads to

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial Z} &= (1-2f)(Z^{-\rho}-1) + f((Z-\eta)^{-\rho}-1)\left(1 - \frac{\partial\eta}{\partial Z}\right)\\ &\quad + f((Z+\eta)^{-\rho}-1)\left(1+\frac{\partial\eta}{\partial Z}\right) + f(\alpha_H - \alpha_L)\frac{\partial\chi}{\partial Z}\\ &= (1-2f)(Z^{-\rho}-1) + \frac{2f}{1+\lambda}\left(\left(\frac{2}{1+\lambda}\right)^{-\rho} Z^{-\rho} - 1\right)\\ &\quad + \frac{2f\lambda}{1+\lambda}\left(\left(\frac{2\lambda}{1+\lambda}\right)^{-\rho} Z^{-\rho} - 1\right) + f(\alpha_H - \alpha_L)\left(\left(\frac{2\lambda}{1+\lambda}\right)^{1-\rho} - 1\right)\frac{Z^{-\rho}}{\alpha_L},\end{aligned}$$

where the second equation is derived based on following properties in Region 1. $Z + \eta = \lambda(Z - \eta)$, $1 - \partial\eta/\partial Z = 2/(1+\lambda)$, $1 + \partial\eta/\partial Z = 2\lambda/(1+\lambda)$, $Z - \eta = 2Z/(1+\lambda)$, and

$$\frac{\partial\chi}{\partial Z} = \frac{Z^{-\rho}}{\alpha_L}\left(\left(\frac{2\lambda}{1+\lambda}\right)^{1-\rho} - 1\right).$$

First, $\lim_{\mu\to\mu_3}(\partial\mathcal{W}/\partial\mu) < 0$ or $\lim_{Z\to\hat{q}}(\partial\mathcal{W}/\partial Z) > 0$ easily follows from the fact that $\lim_{Z\to\hat{q}}(\partial\chi/\partial Z) = 0$. Next,

$$\lim_{Z\to(q^*+\bar{\bar{q}})/2}\frac{\partial\mathcal{W}}{\partial Z} = (1-2f)\left(\left(\frac{1+\lambda}{2\lambda}\right)^{-\rho}-1\right)+\frac{2f}{1+\lambda}\left(\lambda^{-\rho}-1\right)$$
$$+\frac{f(\alpha_H-\alpha_L)}{\alpha_L}\left(\frac{1+\lambda}{2\lambda}\right)^{-\rho}\left(\left(\frac{2\lambda}{1+\lambda}\right)^{1-\rho}-1\right)$$
$$=\left(\frac{2\lambda}{1+\lambda}\right)^{\rho}\left(1-2f-\frac{f(\alpha_H-\alpha_L)}{\alpha_L}\right)+\frac{2f(\alpha_H-\alpha_L)}{\alpha_L}-(1-2f)<0.$$

The above inequality is true if condition 1, 2, and 3 are true. This proves why $\lim_{\mu\to\mu_2}(\partial\mathcal{W}/\partial\mu) > 0$ under the condition 1, 2, and 3. Finally, to prove $\partial^2\mathcal{W}/\partial\mu^2 < 0$ it suffices to show $\partial^2\mathcal{W}/\partial Z^2$ is monotone in Z, which is given below.

$$\frac{\partial^2\mathcal{W}}{\partial Z^2} = -\rho(1-2f)Z^{-\rho-1}-\frac{2f\rho}{1+\lambda}\left(\frac{2}{1+\lambda}\right)^{-\rho}Z^{-\rho-1}-\frac{2f\rho\lambda}{1+\lambda}\left(\frac{2\lambda}{1+\lambda}\right)^{-\rho}Z^{-\rho-1}$$
$$-\rho f(\alpha_H-\alpha_L)\left(\left(\frac{2\lambda}{1+\lambda}\right)^{1-\rho}-1\right)\frac{Z^{-\rho-1}}{\alpha_L}.$$

As for the equilibrium Region 2 with $\rho > 1$, eq.(8) leads to

$$\frac{\partial\mathcal{W}}{\partial Z} = (1-2f)(Z^{-\rho}-1)+f(\alpha_H-\alpha_L)\frac{\partial\chi}{\partial Z}$$
$$=(1-2f)(Z^{-\rho}-1)+f(\alpha_H-\alpha_L)\frac{2-Z^{-\rho}}{\alpha_L},$$

which could be non-positive only if $Z < 2^{-1/\rho}$, $f(\alpha_H-\alpha_L)/\alpha_L > 1-2f$, and $Z \le G(f,\alpha_H,\alpha_L)^{-1/\rho}$. Since $f < 0.5$, $G(f,\alpha_H,\alpha_L) > 2$. Thus, $2^{-1/\rho} > G(f,\alpha_H,\alpha_L)^{-1/\rho}$. So one needs to satisfy only $Z \le G(f,\alpha_H,\alpha_L)^{-1/\rho}$ and $f(\alpha_H-\alpha_L)/\alpha_L > 1-2f$ to show $\partial\mathcal{W}/\partial Z \le 0$. The lower bound for Z in Region 2 equals $(1+\lambda^{-1})/2$. Hence, as long as $(1+\lambda^{-1})/2 < G(f,\alpha_H,\alpha_L)^{-1/\rho}$, $\partial\mathcal{W}/\partial Z \le 0$ under $\mu \ge \mu_1$. The former condition is exactly identical as the sufficient condition 3. This completes the proof. ■

A.4 Perfectly Competitive secondary asset market

In this section we explore the possibility that the secondary asset market is not characterized by bilateral meetings and bargaining, but we assume that it is perfectly competitive.

Under this assumption, the OTC value function for $H-$type is given by

$$\begin{aligned}\Omega_{H}(m) = \max_{\delta^s} &\{V_{H}(m-\delta^s, a+\delta^s/S)\} \\ \text{s.t. } &\chi^d = \delta^s/S, \\ &\delta^s \leq m, \end{aligned} \tag{a.11}$$

where S is the OTC price of asset in units of money. Similarly, $L-$type's OTC value function is given by

$$\begin{aligned}\Omega_{L}(\tilde{m}) = \max_{\chi^s} &\{V_{L}(\tilde{m}+S\chi^s, a-\chi)\} \\ \text{s.t. } &\delta^d = S\chi^s, \\ &\chi^s \leq a, \end{aligned} \tag{a.12}$$

First order conditions with respect to δ^s and χ^s are given respectively by:

$$\begin{aligned} u'(q_{H})\frac{\partial q_{H}}{\partial m} + \varphi\left(1 - \frac{\partial p_{H}(m-\delta^s)}{\partial m}\right) + \lambda =& \frac{\alpha_{H}}{S} \\ u'(q_{L})\frac{\partial q_{L}}{\partial \tilde{m}} + \varphi\left(1 - \frac{\partial p_{L}(\tilde{m}+S\chi^s)}{\partial \tilde{m}}\right) S =& \alpha_{L} + \eta, \end{aligned}$$

where λ and η are Lagrangian multiplier with respect to a.11 and a.12. We then analyze each case separately.

Case 1: $\lambda = 0$ and $\eta = 0$.

Sub-case 1.1: $m - \delta^s \geq m^*$ and $m - \delta^d \geq m^*$

In this case, FOCs imply that $\varphi = \alpha_{H}/S$ and $S\varphi = \alpha_{L}$, hence contradiction.

Sub-case 1.2: $\tilde{m} + \delta^s < m^*$ and $m - \delta^d \geq m^*$

In this case, FOCs imply that $\delta^s = m - \bar{m}$, $\delta^d = \frac{\ell}{1-\ell}\delta^s$, $\chi^d = (m-\bar{m})/S$, $\chi^s = \frac{\ell}{1-\ell}\frac{m-\bar{m}}{S}$, $\frac{\ell}{1-\ell}\varphi(m-\bar{m})u'(\varphi\bar{m}) < \alpha_{H}A$, and $S = \frac{\alpha_H}{\varphi u'(\varphi\bar{m})}$.

Sub-case 1.3: $m - \delta^s < m^*$ and $\tilde{m} + \delta^d \geq m^*$

FOCs imply that δ^s is such that $\frac{u'(\varphi(m-\delta^s))}{u'(\varphi(\tilde{m}+\frac{\ell}{1-\ell}\delta^s))} = \frac{\alpha_H}{\alpha_L}$. $\delta^d = \frac{\ell}{1-\ell}\delta^s$, $\chi^d = \delta^s/S$, $\chi^s = \frac{\ell}{1-\ell}\frac{\delta^s}{S}$, $S = \frac{\alpha_H}{\varphi u'(\varphi(m-\delta^s))}$. In this case, the following two inequalities also hold true. $\ell m + (1-\ell)\tilde{m} < (1-\ell)m^* + \ell\bar{m}$. $\frac{\ell}{1-\ell}\varphi\delta^s u'(\varphi(m-\delta^s)) < \alpha_{H} a$.

Case 2: $\lambda > 0$ and $\eta = 0$.

In this case, FOCs say $u'(0) + \varphi + \lambda = \alpha_{H}/S = \infty$, hence contradiction.

Case 3: $\lambda = 0$ and $\eta > 0$.

Sub-case 3.1: $m - \delta^s \geq m^*$ and $\tilde{m} + \delta^d \geq m^*$

FOCs imply that $\chi^d = \frac{1-\ell}{\ell}a$, $\chi^s = a$, $\delta^s = \frac{1-\ell}{\ell}\frac{\alpha_H a}{\varphi}$, $\delta^d = \frac{\alpha_H a}{\varphi}$, $S = \alpha_{H}/\varphi$. In this case, the

following two inequalities also hold true. $\alpha_H a < \frac{\ell}{1-\ell}\varphi m$. and $\ell m+(1-\ell)\tilde{m} > (1-\ell)m^* + \ell\bar{m}$.
Sub-case 3.2: $m - \delta^s < m^*$ and $\tilde{m} + \delta^d \geq m^*$

FOCs imply that $\chi^d = \frac{1-\ell}{\ell}a$, $\chi^s = a$, $\delta^s = \frac{1-\ell}{\ell}\delta^d$, $\delta^d = Sa$, $S = \alpha_H/\varphi u'(\varphi(m-\delta^s))$. In this case, the following two inequalities also hold true. $\alpha_H a \leq \frac{\ell}{1-\ell}\varphi(m-\bar{m})u'(\varphi(m-\delta^s))$. and $\ell m + (1-\ell)\tilde{m} > (1-\ell)m^* + \ell\bar{m}$.
Sub-case 3.3: $m - \delta^s < m^*$ and $\tilde{m} + \delta^d \geq m^*$

FOCs imply that $\chi^d = \frac{1-\ell}{\ell}a$, $\chi^s = a$, $\delta^s = \frac{1-\ell}{\ell}\delta^d$, $\delta^d = Sa$, $S = \alpha_H/\varphi u'(\varphi(m-\delta^s))$. In this case, the following two inequalities also hold true. $\alpha_H a \leq \frac{\ell}{1-\ell}\varphi m u'(\varphi(m-\delta^s))$. and $\ell m + (1-\ell)\tilde{m} < (1-\ell)m^* + \ell\bar{m}$.
Case 4: $\lambda > 0$ and $\eta > 0$. Here, FOCs say $u'(0)+\varphi+\lambda = \alpha_H/S = \infty$, hence contradiction.

The following lemma summarizes OTC solutions under perfect competition

Lemma 4 *Define the cutoff level of asset holdings.*

$$\bar{a}(m,\tilde{m}) = \begin{cases} \frac{\ell\varphi(m-\bar{m})u'(\varphi\bar{m})}{\alpha_H(1-\ell)} & \text{if } \ell m + (1-\ell)\tilde{m} \geq (1-\ell)m^* + \ell\bar{m}, \\ \frac{\ell\varphi\delta^s(m,\tilde{m})u'(\varphi(m-\delta^s(m,\tilde{m})))}{\alpha_H(1-\ell)} & \text{if } \ell m + (1-\ell)\tilde{m} < (1-\ell)m^* + \ell\bar{m}, \end{cases}$$

Then the solution to the bargaining problem is given by

$$\begin{aligned}
\chi^d(m,\tilde{m}) &= \begin{cases} \frac{\lambda^s(m,\tilde{m})}{S} & \text{if } a \geq \bar{a}(m,\tilde{m}), \\ \frac{1-\ell}{\ell}a & \text{if } a < \bar{a}(m,\tilde{m}), \end{cases} \\
\chi^s &= \frac{\ell}{1-\ell}\chi^d \\
S &= \begin{cases} \frac{\alpha_H}{\varphi u'(\varphi(m-\delta^s(m,\tilde{m})))} & \text{if } a \geq \bar{a}(m,\tilde{m}), \\ \frac{\alpha_H}{\varphi u'(\varphi(m-\Delta^s(m)))} & \text{if } a < \bar{a}(m,\tilde{m}), \end{cases} \\
\delta^s(m,\tilde{m}) &= \begin{cases} m-\bar{m} & \text{if } \ell m + (1-\ell)\tilde{m} \geq (1-\ell)m^* + \ell\bar{m}, \\ \left\{\delta : \frac{u'(\varphi(m-\delta))}{u'(\varphi(\tilde{m}+\frac{\ell}{1-\ell}\delta))} = \frac{\alpha_H}{\alpha_L}\right\} & \text{if } \ell m + (1-\ell)\tilde{m} < (1-\ell)m^* + \ell\bar{m}, \end{cases} \\
\delta^d(m,\tilde{m}) &= \frac{\ell}{1-\ell}\delta^s(m,\tilde{m}) \\
\Delta^s(m) &= \left\{\Delta : \Delta = a\frac{1-\ell}{\ell}\frac{\alpha_H}{\varphi u'(\varphi(m-\Delta))}\right\}, \\
\Delta^d(m) &= \frac{\ell}{1-\ell}\Delta^s(m).
\end{aligned}$$

Therefore, OTC outcomes under the perfect competition look similar to Figure 1 too. In other words, OTC solutions are qualitatively similar to those under bilateral bargaining protocols. This implies our novel welfare channel will still hold true in equilibrium.

For instance, assuming $\ell = 0.5$, steady-state effects of change in real balances Z on asset transfer χ will be similar to Proposition 1. Specifically, under equilibrium Region 1, $\chi = \frac{1}{\alpha_H}(\frac{1-\theta}{1+\theta})^{-\rho}(\frac{2\theta}{1+\theta})^{-\rho}Z^{1-\rho}$. Hence, the following should be true.

$$\frac{\partial \chi}{\partial Z} = \begin{cases} < 0, & \text{if } \rho > 1, \\ = 0, & \text{if } \rho = 1, \\ > 0, & \text{if } \rho < 1, \end{cases} \quad \frac{\partial^2 \chi}{\partial Z^2} = \begin{cases} > 0, & \text{if } \rho > 1, \\ = 0, & \text{if } \rho = 1, \\ < 0, & \text{if } \rho < 1, \end{cases}$$

which is the same result as in Proposition 1. Under equilibrium Region 2, $\chi = \frac{1}{\alpha_H}(Z - \bar{Z})u'(\bar{Z})$. Hence, $\partial\chi/\partial Z > 0$ always, which is different from Proposition 1. But, as we show in the welfare section, welfare improving inflation happens under equilibrium Region 1. Thus, the welfare analysis is qualitatively similar even with a perfectly competitive secondary asset market.

The intuition is quite simple. In our model, welfare can be increasing in inflation because a higher inflation makes L-types more desperate to acquire money (by selling assets), thus promoting beneficial asset trades in the secondary asset market. While the structure of that secondary asset market is paramount for how efficiently agents will meet with each other (search frictions versus Walrasian trade) and how they will split the surplus (bilateral bargaining versus perfect competition), it does not change the fact that higher inflation will make L-types more eager to trade. Details aside, this will increase the volume of trade in the secondary asset market, which is beneficial for welfare.