The Liquidity-Augmented Model of Macroeconomic Aggregates

Athanasios Geromichalos
University of California – Davis

Lucas Herrenbrueck
Simon Fraser University

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ABSTRACT

We propose a new model of the macroeconomy which is simple and tractable, yet explicit about the foundations of liquidity. Monetary policy is implemented via swaps of money for liquid bonds in a secondary asset market. Prices are flexible, yet policy has real effects because money, bonds, and capital are imperfect substitutes, both in the short run and in the long run. The model unifies two classical channels through which the price of liquidity affects the economy (Friedman's real balance effect vs Mundell's and Tobin's asset substitution effect), and it shines light on important macroeconomic questions: the causal link between interest rates and inflation, the effects of money demand shocks, and the existence and persistence of a liquidity trap where interest rates are zero but inflation is positive.

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Email: ageromich@ucdavis.edu, herrenbrueck@sfu.ca. Declarations of interest: none.

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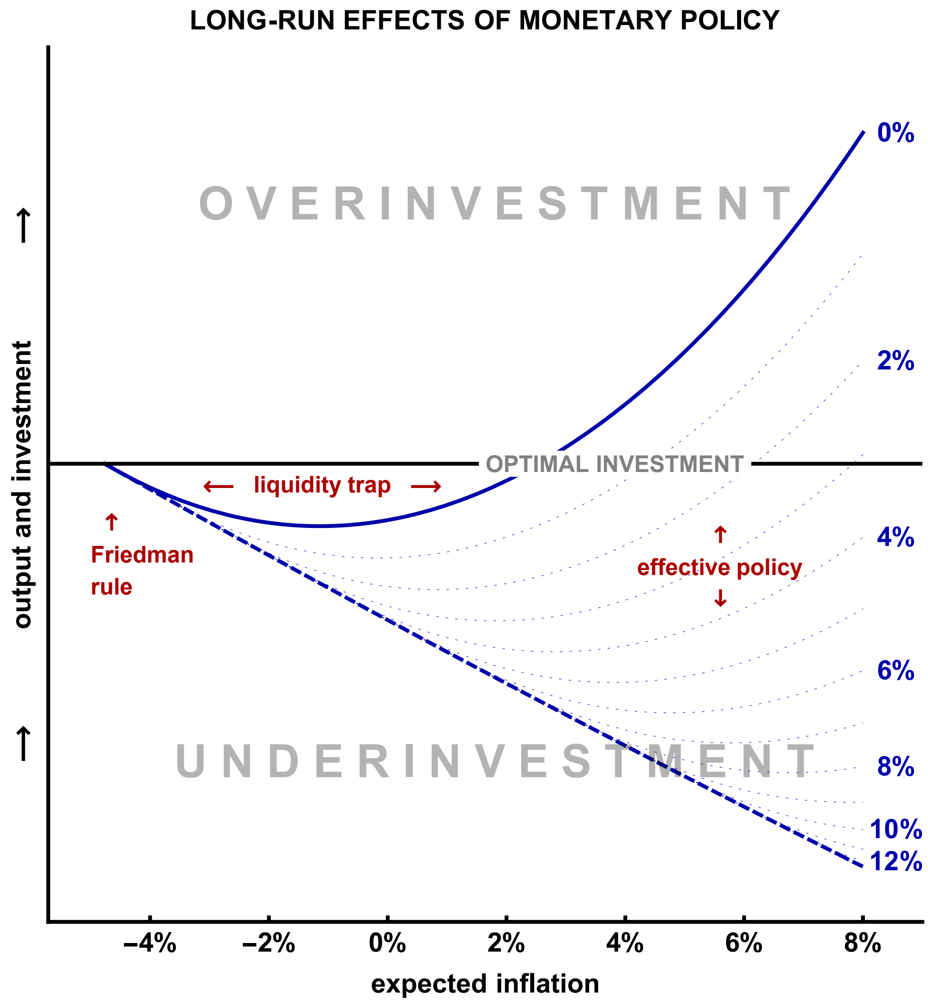
Some questions about the paper and model have come up repeatedly. To do them justice, we answer them in an FAQ here (link). Code used for estimates and simulation can be found here (link).

GRAPHICAL ABSTRACT



1 Introduction

We propose a model for macroeconomic analysis that is parsimonious, tractable, consistent with the microfoundations of asset liquidity, and also consistent with a set of facts that have been challenging to model in a unified framework: (1) monetary policy is implemented via intervention in financial markets; (2) few assets serve as media of exchange ("money") but most assets can be sold when needed, thus acting as *indirect* substitutes to money; (3) lower policy interest rates tend to increase output and investment; and (4) there is no tight link between policy interest rates and inflation, neither in the short run nor in the long run. The model can serve as a framework to study many topics in macro- and monetary economics. Here, to demonstrate the breadth of possible applications, we use it to address the following questions: the short-term effects of money demand shocks, the links between inflation, interest rates, and asset returns, optimal monetary policy in the long run, and the existence and persistence of a liquidity trap where interest rates are zero but inflation is positive.

Why is such a model needed? The model which has been most widely used as a guide to policy, the New Keynesian model, features a cashless economy where the driving friction is price stickiness. As a result, the model is not well suited to modeling monetary issues such as money demand shocks, or intervention in financial markets (it is cashless); moreover, its ability to explain a liquidity trap has been challenged.[1] These issues are easier to address in a New Monetarist model where the driving frictions make liquidity emerge naturally (Lagos, Rocheteau, and Wright, 2017). However, that branch of the literature has mostly focused on inflation and the real balance effect, at the expense of a realistic model of interest rates, their central role in monetary policy, and their effect on the economy.

Our model combines New Monetarist insights about asset liquidity with the structure of the neoclassical growth model – the main workhorse model of macroeconomic aggregates. Hence, we call it the Liquidity-Augmented Model of Macroeconomic Aggregates (LAMMA). Due to frictions that we will describe precisely, a need for a medium of exchange arises in the economy, and in the model this role is played by fiat money. Government bonds and physical capital cannot be used as media of exchange, but they too are liquid, as agents with a need for money can sell their bonds and capital in a secondary asset market. The government controls the quantities of money and liquid bonds, and can therefore conduct open-market operations in that secondary market to target the price of liquid bonds, which is arguably the empirically relevant approach.

Our first result is that monetary policy has real effects at all frequencies, even in steady state. We can express the long-run effects as a function of two rates: the expected inflation rate, and the interest rate on liquid bonds. Expected inflation makes people economize on money balances, which gives rise to two opposing forces. One is the inflation tax which falls

[1] See Benhabib, Schmitt-Grohé, and Uribe (2001), Bullard (2015), and Cochrane (2017).

on the productive economy, and this force tends to make money and capital complements in general equilibrium. The second force is the fact that as a somewhat liquid asset, capital can be an imperfect substitute to money (the Mundell-Tobin effect). The end result could be overinvestment or underinvestment, and which force prevails depends on the second instrument of monetary policy: the interest rate on bonds. Raising this rate by selling bonds in the asset market makes bonds a more desirable store of value, hence investment and output fall. Lowering this rate, by buying bonds in the asset market, does the opposite. With the right interest rate, investment and output are at their first-best levels; hence, while the Friedman rule is *an* optimal long-run policy in this economy, it is not the *only* such policy.[2]

Second, the model clarifies that the distinction between interest rates on liquid and illiquid assets is crucial for understanding the role of monetary policy, and for empirical analysis of its effects. Highly liquid assets are the closest substitutes for money, hence their returns can be set independently from inflation, even in the long run. For this reason, we should not expect there to be an empirically stable, structural Fisher equation for less-than-perfectly-illiquid assets. Depending on the monetary policy regime and the trends affecting the economy, the empirical slope of inflation on interest rates can be greater than one, less than one, or even negative. Using long-run data, we demonstrate that this effect holds for capital assets as well; inflation and interest rates affect capital returns separately, and with coefficients close to those predicted by the model.

When capital is hard to trade, the economy can be in a liquidity trap, which we define as a situation where (i) the policy interest rate is at a lower bound, (ii) output and investment are below their optimal levels, and (iii) raising interest rates would make things worse (equivalent to saying that it would be desirable to lower interest rates further). This liquidity trap formalizes the long-held notion that saving is not automatically translated into investment, but requires a well-functioning financial system and an unconstrained interest rate. In such a trap, a variety of fiscal schemes may help, but there is also a simple monetary remedy: increase inflation permanently. In addition, there is nothing "short-run" about our mechanism; hence, there is no contradiction between a liquidity trap and stable, even positive, inflation. This fits with the experience of developed economies in the last decade (three decades in Japan), where near-zero interest rates have coexisted with stable inflation.

Finally, we solve our model in the short run as a dynamic-stochastic general equilibrium model. We discuss various schemes of monetary policy implementation, but focus on the most relevant one for the modern era: interest-rate setting monetary policy, implemented via intervention in secondary asset markets, together with endogenous money growth. Next we calibrate the model and simulate the economy's response to several real and financial shocks, including a shock to the fraction of agents who need money to pay for goods (that is to say,

[2]There is also a set of parameters where the comparative statics described above are reversed, so that raising interest rates on liquid bonds stimulates investment and output.

a money demand shock); we show that the shocks have plausible effects on macroeconomic outcomes. It is worth emphasizing that the results discussed so far are representative of the topics that the LAMMA can adress, but by no means exhaustive. One contribution of the paper is to offer a *framework* where researchers can study monetary policy in a theoretically consistent and empirically relevant way, while at the same time allowing them to include any feature that one could have included in the neoclassical growth model itself.

Conceptually, our paper is related to Tobin (1969). Writing in the inaugural issue of the *Journal of Money, Credit and Banking*, he proposes a "general framework for monetary analysis":

> "Monetary policy can be introduced by allowing some government debt to take non-monetary form. Then, even though total government debt is fixed [. . .], its composition can be altered by open market operations. [. . .] It is assumed [that money, bonds and capital] are gross substitutes; the demand for each asset varies directly with its own rate and inversely with other rates."

Although today we can do better than "it is assumed", there is no doubt that Tobin's model contains the right ingredients: money, bonds, and capital.[3] Our model contains the same ingredients, but provides microfoundations of why these assets are liquid and how monetary policy can exploit their relationship and affect the macroeconomy. In order to give meaning to "monetary", we are explicit about the frictions that make monetary trade emerge. In order to give meaning to "monetary policy", we add bonds that are endogenously imperfect substitutes to money, and a financial market where the monetary authority intervenes to "set interest rates". Finally, in order to capture the effects of monetary policy in a realistic way, our crucial addition is to recognize the *dual role of capital*: it is useful in production, as in the neoclassical model, and it can be traded (at least sometimes) in financial markets, making it liquid and making its yield integrated with the yields on other liquid assets.

Our paper is part of a literature that studies how liquidity and monetary policy can shape asset prices, based on the New Monetarist paradigm (Lagos and Wright, 2005; Lagos et al., 2017). In papers like Geromichalos, Licari, and Suárez-Lledó (2007), Lester, Postlewaite, and Wright (2012), Nosal and Rocheteau (2013), Andolfatto and Martin (2013), and Hu and Rocheteau (2015), assets are 'liquid' in the sense that they serve directly as media of exchange (often alongside money).[4] An alternative approach highlights that assets may be priced at a liquidity premium not because they serve as media of exchange (an assumption often defied by real-world observation), but because agents can sell them for money when they need

[3] Also inspired by Tobin, Andres, López-Salido, and Nelson (2004) develop a model with imperfect substitutability between money and bonds and study the effect of monetary policy on long-term asset returns. Unlike us, they do not include capital in their model.

[4] Some papers in this literature revisit well-known asset pricing puzzles and suggest that asset liquidity may be the key to rationalizing these puzzles. Examples include Lagos (2010), Geromichalos and Simonovska (2014), Lagos and Zhang (2015), and Geromichalos, Herrenbrueck, and Salyer (2016).

it (Geromichalos and Herrenbrueck, 2016; Berentsen, Huber, and Marchesiani, 2014, 2016; Mattesini and Nosal, 2016).[5] Herrenbrueck and Geromichalos (2017) dub this alternative approach *indirect* liquidity. In this paper, we make use of the indirect liquidity approach because it provides a natural way to mimic how central banks implement monetary policy in reality: they intervene in institutions where agents trade assets in response to short-term liquidity needs. That is exactly what the secondary asset market in our model represents.

One central question for us is the effect of monetary policy on capital, and we have argued that the dual role of capital, as a productive factor (affected by the inflation tax) and a liquid asset (competing with money as a store of value), is key to this (see also Herrenbrueck, 2014). Most of the recent literature has focused on one of these channels at a time. Aruoba, Waller, and Wright (2011) analyze how capital responds negatively to the inflation tax. In Rocheteau, Wright, and Zhang (2018), entrepreneurs can finance investment using money or credit, thus inflation also tends to depress investment. On the other hand, Lagos and Rocheteau (2008), Rocheteau and Rodriguez-Lopez (2014), and Venkateswaran and Wright (2014) explore the idea that capital could be valued for its potential liquidity properties as a substitute to money, which makes inflation cause *over*accumulation of capital unless offset by a negative externality or capital tax.[6]

Our final important departure from most of the New Monetarist literature is that we interpret the yield on liquid bonds as the main monetary policy instrument. Traditionally, this literature has emphasized money growth and the real balance effect of expected inflation. The list of papers that do focus on the yield on liquid bonds is growing and now includes Andolfatto and Williamson (2015), Dong and Xiao (2017), Rocheteau, Wright, and Xiao (2018), and Dominguez and Gomis-Porqueras (2019). These papers do not include capital or the effect of monetary policy on investment.

Our paper is also related to a large literature on the effect of monetary policy on macroeconomic aggregates in the presence of financial frictions. Notable examples include Bernanke and Gertler (1989), Cúrdia and Woodford (2011), and in particular Kiyotaki and Moore (2019) who also consider a multi-asset model where money has superior liquidity relative to other assets. Unlike us, they do not explicitly model bonds (and, hence, the conduct of monetary policy through intervention in the market for these bonds). Finally, our paper is related to

[5]In these papers, agents who receive an idiosyncratic consumption opportunity visit a market where they can sell financial assets and acquire money from agents who do not need it as badly. This idea is related to Berentsen, Camera, and Waller (2007), where the allocation of money into the hands of the agents who need it the most takes place through a (frictionless) banking system rather than through secondary asset markets.

[6]Empirical evidence does not resolve the question which one of the two effects dominates. First, the evidence that exists is ambiguous: in the long run, inflation seems to be positively related to investment at low levels, but negatively at higher levels (Bullard and Keating, 1995; Bullard, 1999); positively in the U.S. time series (Ahmed and Rogers, 2000), but negatively in the OECD cross-section (Madsen, 2003). Second, there is a strong theoretical reason why the evidence *should* be ambiguous; as we show, an optimal monetary policy *makes* investment unrelated to inflation in the long run. In other words, monetary policy works because it can exploit the Mundell-Tobin effect, but if this is done optimally, empirical evidence of the effect will be obscured.

the literature on policy in a liquidity trap, including Krugman (1998), Eggertsson and Woodford (2003), Andolfatto (2003), Werning (2011), Williamson (2012), Guerrieri and Lorenzoni (2017), Altermatt (2017), and Caramp and Singh (2020).

The paper is organized as follows. Section 2 introduces the model. Section 3 focuses on steady states: we analyze equilibria, discuss policy options, and apply the results to long-run questions such as the liquidity trap and the effect of inflation on capital. In Section 5, we return to the stochastic economy: we calibrate the model and solve it in log-linearized form, compare the effects of various real and financial shocks, and discuss short-term policy options. Section 6 concludes, and additional details are provided in the Appendix.

2 The model

2.1 Environment

Time $t = 0, 1, \ldots$ is discrete and runs forever. The economy consists of a unit measure of households, an indeterminate measure of firms, and a consolidated government that controls fiscal and monetary policy. (In Appendix A.3, we extend the model to distinguish explicitly between a fiscal and a monetary authority.) Each household has two members: a worker and a shopper, who make decisions jointly to maximize the household's utility. The economy is subject to information and commitment frictions: all private agents are anonymous, therefore they cannot make long-term promises, and all trade must be quid-pro-quo.

Each period is divided into three sub-periods: an asset market (AM), a production market (PM), and a centralized market (CM). During the PM, shoppers buy goods from firms, and due to anonymity, they must pay for them with a suitable medium of exchange. The firms rent labor and capital from the households, and combine them to produce goods. In the CM, households divide the output goods between consumption and investment. Households also choose their asset portfolios for the next period – hence, the CM is the "primary" asset market. In the next morning, shoppers learn of a random opportunity trade with a firm during the PM. Since such trade requires a medium of exchange, shoppers may want to trade with other households to rebalance their portfolios; they can do so in the AM, which is therefore the "secondary" asset market. Households are active in all three periods; firms are active only during the PM, and the government is only active during the AM and CM subperiods. This timing is illustrated in Figure 1.

There are three assets in the economy: money (in aggregate supply M), nominal discount bonds (B), and physical capital (K). The government controls M and B, whereas capital is created by households through investment. Money is special in that it is the only asset suitable as a medium of exchange in the PM, because it is portable and easily recognized by everyone. (Bonds are book entries and the technology to verify them is prohibitively

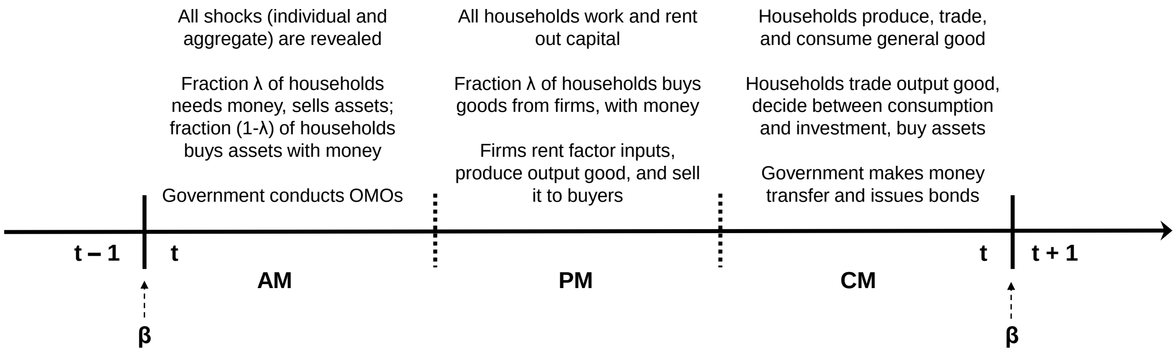


Figure 1: Timing of events.

expensive to be used during the PM. Physical capital is made-for-purpose, not portable, and the technology to verify capital ownership also does not exist in the PM.[7]) Bonds are special in that they are easier to trade than capital during the AM: agents can sell all of their bonds, but only a fraction $\eta_t \in [0,1]$ of their capital.[8] Capital is special in that it is both a tradable asset and a productive input. Hence, money is the most liquid asset: it can be used to purchase anything. Bonds and capital cannot be used to purchase goods, but they can be sold for money when money is needed; thus, they have *indirect* liquidity properties.

During the PM, firms operate a technology that turns capital (k_t) and labor (h_t) into an output good y_t. Firms rent capital and labor from the worker-members of the households, on a competitive factor market (but the factor income arrives too late to be used by the shopper in the same period). The production function is standard:

$$y_t = Ak_t^\alpha h_t^{1-\alpha}$$

Due to anonymity and a lack of a double coincidence of wants, a medium of exchange is required to conduct trade in the PM, and, as already explained, money is the unique object that can serve this role. Additionally, we assume that only a random fraction $\lambda_t \in (0,1)$ of shoppers will enter the PM. Once they are in the PM, trading with firms is competitive. (In

[7]These assumptions are consistent with empirical observation, as is their implication: we rarely see bonds or capital serving directly as means of payment in transactions. There are other potential explanations: a seller may be reluctant to accept a bond or a claim to capital, either because she does not know what they are supposed to look like, because they may be just zeros and ones in a computer, or because they are easy to counterfeit (Lester et al., 2012). Finally, Rocheteau (2011) shows that if there is asymmetric information regarding the future returns of financial assets, then money will arise endogenously as a superior medium of exchange.

[8]This assumption allows us to capture the reasonable idea that capital is less tradable than bonds while maintaining the tractability of our model. A bond delivers one dollar at the end of the current period; thus, any agent who does not have a current need for money will be happy to buy such bonds (at the right price). However, selling a piece of machinery or a building is less straightforward, as one first needs to find the *right* buyer(s) for these items. Hence, one can also think of η as the probability with which a suitable buyer is located.

Appendix A.2, we introduce search frictions and price posting by firms in the goods market, and show that competitive pricing arises as a limiting case when search frictions are small.)

All shocks to period-t variables are revealed at the beginning of that period, before the AM and PM open. Consequently, some shoppers learn that they will trade in the PM during the period, and others learn that they will not. As long as there is a positive cost of holding money, shoppers will never hold enough of it to satiate them in the goods market, but other shoppers will end up with money that they do not need in the same period. Hence, liquidity is misallocated. In order to correct this, shoppers visit the AM: those who need money seek to liquidate assets, while the others use their money to buy assets at a good price.[9] The government can also intervene in the secondary market by selling additional bonds, or by buying bonds with additional money. Pricing is competitive.

During the CM, households can buy or sell any asset, as well as the output good y, on a competitive market. They then choose how much of the output good is to be consumed (c_t), and they choose their asset holdings $(m_{t+1}, b_{t+1}, k_{t+1})$ for the next period. The government makes a nominal lump-sum transfer T_t to all households (a tax if negative), pays out the bond dividends to the households (one unit of money per bond), and issues new bonds. A fraction $\delta \in (0, 1)$ of existing capital depreciates, and it can be replaced by investing some of the available output. Hence, the law of motion of the aggregate capital stock is:

$$k_{t+1} = y_t - c_t + (1 - \delta)k_t$$

Also during the CM, households can produce, consume, and trade a "general" consumption good, $g \in \mathbb{R}$, where we interpret negative values as production and positive ones as consumption. As shown in Lagos (2010), this good is a convenient way to induce linear preferences and thereby collapse the portfolio problem into something tractable. The good has no other function in our paper; in particular, it cannot be used for investment.

Households discount the future at rate $\rho \equiv (1 - \beta)/\beta$, where $\rho > 0$ and/or $\beta < 1$. (Most of the equations in our paper will be more readable in terms of ρ, but we will use β in a few cases where that makes more sense.) Households have the following utility function:

$$U_t(c_t, g_t) = u(c_t) + g_t,$$

where u is a twice continuously differentiable function that satisfies $u' > 0$ and $u'' < 0$. In particular, for some examples we use $u(c) = (c^{1-\sigma} - 1)/(1-\sigma)$, where $\sigma > 0$ is the inverse elasticity of intertemporal substitution. Labor generates no disutility, but a household's worker is only able to supply labor up to an endowment normalized to 1.

[9] An alternative interpretation of our AM would be as a market where agents pledge their assets as collateral in order to obtain a secured (monetary) loan, as is the case in the repo market.

2.2 The social planner's solution

As a benchmark, consider a social planner who is not bound by commitment problems, and can freely transfer resources between agents. As all households and firms are the same, the planner will treat them symmetrically, and solve the following representative-agent problem, choosing a sequence of capital stocks and labor and consumption allocations:

$$\text{(SP)} \qquad W(K_t) = \max_{\substack{K_{t+1},\\ H_t, C_t, G_t}} \left\{ u(C_t) + G_t + \frac{1}{1+\rho}\,\mathbb{E}_t\left\{W(K_{t+1})\right\} \right\}$$

$$\text{subject to:} \quad C_t + K_{t+1} = AK_t^{\alpha}H_t^{1-\alpha} + (1-\delta)K_t, \quad H_t \leq 1, \quad \text{and} \quad G_t = 0$$

The initial capital stock K_0 is taken as given.

As the G-consumption good is in zero net supply, this is equivalent to the well-known neoclassical Ramsey problem. With perfectly inelastic labor supply, we must have $H_t = 1$, thus consumption and the capital stock satisfy:

$$\text{(EE)} \qquad u'(C_t) = \frac{1}{1+\rho}\,\mathbb{E}_t\left\{u'(C_{t+1})\left(\alpha AK_{t+1}^{\alpha-1} + 1 - \delta\right)\right\}$$

$$\text{(LOM)} \qquad C_t + K_{t+1} = AK_t^{\alpha} + (1-\delta)K_t$$

$$\text{(TVC)} \qquad 0 = \lim_{t\to\infty} \frac{u'(C_t)K_t^{\alpha}}{(1+\rho)^t}$$

In steady state, we must have $Y = AK^{\alpha} = C + \delta K$, which we can use to solve:

$$\begin{aligned} Y^* &= A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\rho+\delta}\right)^{\frac{\alpha}{1-\alpha}} & K^* &= \frac{\alpha}{\rho+\delta}\cdot Y^* \\ C^* &= \frac{\rho+(1-\alpha)\delta}{\rho+\delta}\cdot Y^* & H^* &= 1,\ G^* = 0 \end{aligned} \tag{1}$$

2.3 Optimal behavior by private agents

Because of the frictions that the private economy is subject to, the price of output goods in the PM will generally not equal their price in the CM. We denote the ratio between the former and the latter by q; it can also be interpreted as the ratio between the real *purchase price* of output and its subsequent marginal *use value*.

Begin the analysis with firms. Because of constant returns to scale in production, the firms' number is indeterminate, and the representative firm solves the static problem:

$$\text{(FP)} \qquad \max_{Y_t, H_t, K_t} \left\{q_tY_t - w_tH_t - r_tK_t\right\} \quad \text{subject to: } Y_t = AK_t^{\alpha}H_t^{1-\alpha},$$

where w and r are the wage and rental rate on capital, denominated in terms of CM output.

(Hence, they represent the real marginal *revenue* products of labor and capital, which differ from the marginal products by the output price q.) Solving this problem defines demand for labor and capital services:

$$\frac{w_t}{q_t} = A(1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} \qquad\qquad \frac{r_t}{q_t} = A\alpha\left(\frac{K_t}{H_t}\right)^{\alpha-1} \tag{2}$$

Since the supply of labor is capped but its marginal product is positive, we will have $H_t = 1$ in every equilibrium, which pins down the wage. The price of output thus satisfies:

$$q_t = \frac{r_t K_t^{1-\alpha}}{\alpha A} \tag{3}$$

This equation is central for the LAMMA. In steady state, the prices q and r will be determined by Euler equations. Thus, the long-run capital stock is governed by three statistics: productivity, the relative price of output between the PM and the CM, and the marginal revenue product of capital. Monetary policy will affect the long-run economy via q, r, or both.

Households make the dynamic decisions in this economy, thus they have a richer menu of choices which is easiest to describe in stages. Begin with the CM of period t, and consider a household coming in with portfolio (m_t, b_t, k_t, y_t) of money, bonds, capital, and output goods. The household chooses its consumption (c_t and g_t), as well as the asset portfolio $(m_{t+1}, b_{t+1}, k_{t+1})$ to be carried into the next period. The prices of general goods (p_t^G), output goods (P_t), and bonds (p_t^B; all in terms of money) are taken as given, and the transfer of money from the government (T_t) is also taken as given. Since new capital is created by not consuming output goods, the price of capital in the CM will simply be 1 – exactly as it is in the standard neoclassical model.

Let $\Lambda_{t+1} \in \{0, 1\}$ be the random variable indicating whether an individual shopper will be selected to shop in the next period; it is distributed *i.i.d.* with $\mathbb{P}\{\Lambda_{t+1} = 1\} = \lambda_{t+1}$. Letting V^{CM} and V^{AM} denote the value functions in the CM and AM subperiods, respectively, we can describe the household's choice as follows:

$$V^{CM}(m_t, b_t, k_t, y_t) = \max_{\substack{c_t, g_t, m_{t+1},\\ b_{t+1}, k_{t+1}}} \left\{ u(c_t) + g_t + \frac{1}{1+\rho}\,\mathbb{E}_t\left\{V^{AM}(m_{t+1}, b_{t+1}, k_{t+1}, \Lambda_{t+1})\right\} \right\} \tag{4}$$

$$\text{subject to:} \quad P_t(c_t + k_{t+1}) + p_t^G g_t + m_{t+1} + p_t^B b_{t+1} = P_t[y_t + (1-\delta)k_t] + m_t + b_t + T_t$$

At this point, one can confirm that the value function V^{CM} will be linear, and that a household's choice of consumption (c) is independent of its asset portfolio (details are provided in Appendix A.1.1). A household with few assets will work to produce general goods g, and sell them to be able to afford its desired level of c, and its desired future asset portfolio. Conversely, a household with many assets will be consuming general goods.

Working backwards through the period, consider the PM of period t. At this stage, the household decides how much labor (h) and capital services (x) to supply to firms, and how much of the output good y the shopper should buy (if applicable). The household takes factor prices and the price of goods as given. Letting V^{PM} denote the value function in the PM subperiod, we can describe the households' choices as follows:

$$V^{PM}(m_t, b_t, k_t, \Lambda_t) = \max_{y_t, x_t, h_t} \left\{ V^{CM}\left(m_t - P_t(q_t y_t - w_t h_t - r_t x_t), b_t, k_t, y_t\right) \right\}$$
$$\text{subject to:} \quad y_t \leq \Lambda_t \frac{m_t}{P_t q_t}, \quad x_t \leq k_t, \quad \text{and} \quad h_t \leq 1$$

Finally, consider the AM of period t. The shocks Λ_t have just been realized; money is the only asset that can be used to buy goods in the PM, therefore households with $\Lambda_t = 1$ will seek to sell other assets for money, and vice versa. Households can trade any amounts of money and bonds that they own, but they cannot sell them short; to short-sell is to create an asset, and bonds and money are special assets that can only be created by a trusted authority. With capital, households face an additional constraint: only a fraction $\eta_t \in [0, 1]$ can be sold on the market. We denote the amounts of bonds and capital sold by ($\Lambda_t = 1$)-households by (χ_t, ξ_t), respectively. We denote the money spent to buy bonds and capital by the other households by (ζ_t^B, ζ_t^K), respectively. Households take the prices of bonds and capital as given; we denote them by s_t^B and s_t^K, in terms of money.[10] Hence, we can describe the households' choices as follows:

$$V^{AM}(m_t, b_t, k_t, 0) = \max_{\zeta_t^B, \zeta_t^K} \left\{ V^{PM}\left(m_t - \zeta_t^B - \zeta_t^K, b_t + \frac{\zeta_t^B}{s_t^B}, k_t + \frac{\zeta_t^K}{s_t^K}, 0\right) \right\} \tag{5}$$
$$\text{subject to:} \quad \zeta_t^B + \zeta_t^K \leq m_t;$$

$$V^{AM}(m_t, b_t, k_t, 1) = \max_{\chi_t, \xi_t} \left\{ V^{PM}\left(m_t + s_t^B \chi_t + s_t^K \xi_t, b_t - \chi_t, k_t - \xi_t, 1\right) \right\} \tag{6}$$
$$\text{subject to:} \quad \chi_t \leq b_t \quad \text{and} \quad \xi_t \leq \eta_t k_t$$

We relegate the detailed solution of the household's problem to Appendix A.1.1 and only review the highlights here. First, s^B and s^K are linked through a no-arbitrage equation, because the asset buyer (the household with $\Lambda_t = 0$) can choose to spend their money on either bonds or capital and must be indifferent in equilibrium:

$$\frac{s_t^K}{P_t} = (r_t + 1 - \delta)\, s_t^B \tag{7}$$

That is, the real price of capital in the secondary market must equal the price of bonds, times

[10] The letters p and s are intended to be mnemonics for "primary market price" and "secondary market price".

the value of capital in subsequent markets. This value is the real marginal revenue product (r_t), plus the fraction remaining after depreciation $(1-\delta)$.

The solution of the portfolio problem in the primary asset market (the CM) must satisfy the following Euler equations for money, bonds, and capital:

$$\frac{u'(c_t)}{P_t} = \frac{1}{1+\rho}\,\mathbb{E}_t\left\{\frac{u'(c_{t+1})}{P_{t+1}}\left(\lambda_{t+1}\frac{1}{q_{t+1}} + (1-\lambda_{t+1})\frac{1}{s^B_{t+1}}\right)\right\} \tag{8}$$

$$\frac{u'(c_t)}{P_t}\,p^B_t = \frac{1}{1+\rho}\,\mathbb{E}_t\left\{\frac{u'(c_{t+1})}{P_{t+1}}\left(\lambda_{t+1}\frac{s^B_{t+1}}{q_{t+1}} + (1-\lambda_{t+1})\right)\right\} \tag{9}$$

$$u'(c_t) = \frac{1}{1+\rho}\,\mathbb{E}_t\left\{u'(c_{t+1})(r_{t+1}+1-\delta)\left(\lambda_{t+1}\eta_{t+1}\frac{s^B_{t+1}}{q_{t+1}} + (1-\lambda_{t+1}\eta_{t+1})\right)\right\} \tag{10}$$

Naturally, the incentive to accumulate capital depends on conditions in the secondary market, including the liquidity of capital and its resale price s^K. The reason why s^K does not explicitly appear in (10) is that we have substituted it with the secondary market price of bonds, s^B, via Equation (7). We write the Euler equation in this way because monetary policy is implemented via setting the secondary market price of bonds; thus, the equation makes transparent how monetary policy affects the value of capital in the primary market, where investment decisions happen, and how this effect is moderated by the expected tradability of capital in the secondary market (η_{t+1}).

2.4 Government budgets and market clearing

The government controls the supplies of money M_t and liquid bonds B_t, taking the initial values (M_0, B_0) as given. New money can be introduced in two ways: via spending N_t on open-market purchases of existing bonds in the AM, or making T_t in lump-sum transfers in the CM; both N_t and T_t can be negative. New bonds are issued in the CM, where they are sold to the public at the market price. (For now, we assume that these choices are made by a consolidated government, and that B_t refers to bonds *held by the public*; in Appendix A.3, we explicitly distinguish between a monetary authority that controls M_t and N_t and has its own balance sheet, and a fiscal authority that controls B_t and T_t.) The government's choices must satisfy budget and non-negativity constraints, and a no-Ponzi condition:

$$\begin{aligned} &M_{t+1} + p^B_t B_{t+1} + \left(\frac{1}{s^B_t} - 1\right)N_t = M_t + B_t + T_t \qquad \text{for all } t \geq 0; \\ &M_{t+1} \geq 0 \text{ and } B_{t+1} \geq 0 \text{ for all } t \geq 0; \qquad \left\{\frac{B_t}{M_t}\right\}_{t=0}^{\infty} \text{ is bounded} \end{aligned} \tag{11}$$

On the left-hand side of the budget constraint, we find the government's sources of funds in a period: newly issued money and bonds, plus the profits from open-market purchases (since bonds are typically bought at a price less than the redemption value of 1). On the right-hand side, we find the government's liabilities; the existing stocks of money and bonds at the beginning of the period, plus the fiscal transfer to households (which is a tax if $T < 0$).

The market clearing conditions of this economy are as follows, where integrals are to be taken over the measure of all households. In the AM, the demands for bonds and tradable capital must equal their respective supplies (counting the government intervention N_t):

$$\lambda_t \int \chi_t \cdot s_t^B = N_t + (1-\lambda_t) \int \zeta_t^B \qquad \text{and} \qquad \lambda_t \int \xi_t \cdot s_t^K = (1-\lambda_t) \int \zeta_t^K \tag{12}$$

In the PM, where only a fraction λ of households is able to shop, but every household supplies factor services, individual choices must add up to the respective aggregate quantities that solve the firm's problem:

$$\lambda_t \int y_t = Y_t, \qquad h_t = H_t = 1, \qquad x_t = k_t, \qquad \text{and} \qquad \int k_t = K_t \tag{13}$$

And in the CM, demands for goods and assets must equal their respective supplies:

$$\begin{gathered} \int m_{t+1} = M_{t+1}, \qquad \int b_{t+1} = B_{t+1}, \\ \int g_t = 0, \qquad c_t = C_t, \qquad \text{and} \qquad C_t + K_{t+1} = Y_t + (1-\delta)K_t \end{gathered} \tag{14}$$

Since individual households have linear value functions, only the totals of their asset demands have to equal the respective aggregate quantity. But for simplicity, we may as well restrict attention to symmetric asset portfolios at the end of each period.

Definition 1. An *equilibrium* of this economy consists of sequences of quantities $\{c_t, h_t, k_t, Y_t, m_t, b_t, \chi_t, \xi_t, \zeta_t^B, \zeta_t^K\}_{t=0}^{\infty}$ and prices $\{q_t, r_t, w_t, P_t, p_t^B, s_t^B, s_t^K\}_{t=0}^{\infty}$ that satisfy:

- The Euler equations (8)-(10), plus transversality conditions:

$$\lim_{t\to\infty} \frac{u'(c_t)}{(1+\rho)^t P_t} = \lim_{t\to\infty} \frac{u'(c_t)k_t}{(1+\rho)^t} = 0$$

- The firm optimality conditions (2)-(3), and constraints on government policy (11);
- No-arbitrage in the AM (7), and market clearing in the AM (12), PM (13), and CM (14).

An *equilibrium* is said to be *monetary* if $P_t < \infty$ for all $t \geq 0$.

For the rest of the paper, we focus on monetary equilibria.

2.5 Monetary policy: quantities or interest rates? And which interest rate?

The government controls the quantities of two assets: money and tradable bonds. In equilibrium, these quantities imply particular asset prices, or interest rates. Hence, we can choose to define government policy in terms of asset quantities (and growth rates), or in terms of interest and inflation rates that the government is targeting. Each approach has advantages, but the second one is arguably more relevant, since monetary policy since the 1980s has indeed been conducted and communicated in terms of a policy interest rate rather than a money growth rule. For this reason, we proceed by solving our model in terms of interest rates, letting the corresponding asset quantities (M_t, N_t, B_t) adjust in the background. (However, these quantities still show up explicitly in various equations, hence solving the model with different assumptions about policy is straightforward; see Section 5.3 and Appendix A.1.3.)

In particular, since monetary policy in our model is implemented via intervention in the secondary asset market, the secondary market price of bonds is the natural choice of policy instrument in our model. Consider:

> "The effective federal funds rate is the interest rate at which depository institutions ... borrow from and lend to each other overnight to meet short-term business needs."
>
> (Source: https://www.federalreserve.gov/aboutthefed/files/pf_3.pdf)

Or:

> "The Bank [of Canada] carries out monetary policy by ... raising and lowering the target for the overnight rate. The overnight rate is the interest rate at which major financial institutions borrow and lend one-day (or 'overnight') funds among themselves; the Bank sets a target level for that rate. This target for the overnight rate is often referred to as the Bank's policy interest rate."
>
> (Source: http://www.bankofcanada.ca/core-functions/monetary-policy/key-interest-rate/)

Institutional details aside, this is exactly what is going on here: the secondary market is where agents with "short-term business needs" meet to reallocate liquidity "overnight" by trading liquid bonds for money. Exploiting the standard formula that links the price and interest rate of an asset, we thus define the **policy interest rate** to be:

$$1 + j_t \equiv \frac{1}{s_t^B}$$

Since j_t is the yield on a liquid bond, it will typically include a liquidity premium and therefore not obey the (classical form of the) Fisher equation. To illustrate this point more clearly, we contrast j_t with the interest rate that *does* obey the classical Fisher equation; this would be the return on a bond that is nominal, one hundred percent default-free, but one hundred percent *illiquid*, in the sense that it must be held to maturity. Specifically, in our context, imagine a one-period discount bond that is sold in the CM and pays of one unit of money in the subsequent CM. Call its interest rate i_t; in any monetary equilibrium, it must satisfy:

$$1 + i_t \equiv \frac{u'(c_t)/P_t}{\frac{1}{1+\rho}\,\mathbb{E}_t\left\{u'(c_{t+1})/P_{t+1}\right\}} \tag{15}$$

Or, on a balanced growth path where the price level grows at (net) rate π, real consumption grows at rate γ, and the elasticity of intertemporal substitution is $1/\sigma$:

$$1 + i = (1+\rho)(1+\pi)(1+\gamma)^{\sigma} \tag{16}$$

Since this is the Fisher equation, we call i_t the **Fisher interest rate**. In nearly all of macroeconomic theory, this interest rate is taken as the model counterpart to the real-world policy rate (or something very near to it, like the return on short-term government bills).[11]

However, this approach has serious flaws, for both empirical and theoretical reasons. First, it is well-known that Equation (15) fits the data poorly – when the left-hand side is identified with the yield on a short-term, quite liquid asset such as Federal Funds or T-bills (e.g., Hansen and Singleton, 1982; Canzoneri, Cumby, and Diba, 2007; to cite just a few). Viewed through our model, this result is not too surprising because the bond priced by i_t – short-term, perfectly safe, yet perfectly illiquid – does not exist in the real world.[12] Hence, the Fisher interest rate is an abstract object that must be estimated rather than simply observed, just like "the general price level" or "total factor productivity". Based on the fact that the long-run Fisher rate equals expected inflation plus the time discount rate (plus a consumption growth term if the economy is on a balanced growth path), Herrenbrueck (2019b) pro-

[11]To our knowledge, our model is the first to explicitly consider the secondary market interest rate on short-term, liquid bonds to be the main instrument of monetary policy. Andolfatto and Williamson (2015) and Rocheteau et al. (2018) are the closest to us on this count: they do not model a secondary market, but study the yield on liquid bonds in the primary market. Berentsen and Waller (2011), like us, model central bank bank intervention in a secondary market for liquid bonds, but with the goal of price level targeting. The vast majority of the New Monetarist literature uses the money growth rate (or the Fisher interest rate, which has a one-for-one relationship with money growth) as the only monetary policy instrument, and its principal influence on the economy comes through the inflation tax. Some New Monetarist papers also consider the quantity of liquid bonds as a secondary policy tool (e.g. Williamson, 2012; Geromichalos and Herrenbrueck, 2016; Huber and Kim, 2017; Herrenbrueck, 2019a). The New Keynesian model is *also* written in terms of the Fisher rate i_t, which gets defined to be the monetary policy instrument and endowed with real effects on the economy via the assumption of sticky prices, and Equation (15) becomes the "New Keynesian IS curve" (Woodford, 2003).

[12]Safe assets tend to be more liquid (Lagos, 2010; Geromichalos, Herrenbrueck, and Lee, 2018), and short term assets also tend to be more liquid (Geromichalos et al., 2016). To be really precise: when we say that an illiquid bond is one that has to be held to maturity and cannot be traded in between, we actually require that this maturity is so far off that the owner does not anticipate any particular liquidity need that the bond payout could be used for. For example, a 1-month bond cannot be terribly illiquid *by its very nature*; many unanticipated expenditures can be put off for a month or two, or paid for by dipping into a credit line, and then the bond payout can be used to pay off the loan. In accounting, assets of maturity shorter than one year are even considered to be "cash equivalents", for this very reason.

vides such an estimate for the U.S. and shows that it indeed behaves very differently from the T-bill rate, even in the long run.

Of course, no model fits the data perfectly, but the problems with i_t as the policy rate go beyond empirical fit. As the rest of this paper will show, the two rates have quite different effect on equilibrium outcomes, and not just quantitatively: the long-run comparative statics of interest rates (holding inflation fixed) and inflation (holding interest rates fixed) can even point in opposite directions (see Equation (18) and Figure 2).

How is a particular policy rate implemented in our model? Through open-market operations in the secondary asset market. The following equation, derived in detail in Appendix A.1.2, explicitly links the intervention N_t (value of bond purchases if positive, money spent on bond sales if negative) with the policy rate j_t:

$$1 + \mathbf{j_t} = \frac{\lambda_t \left[B_t + P_t(r_t + 1 - \delta)\eta_t K_t\right]}{(1 - \lambda_t)M_t + \mathbf{N_t}} \tag{17}$$

The equation also shows how the intervention needed to achieve a given policy rate depends on state variables (M_t, B_t, K_t), shocks (λ_t, η_t), and the general price level P_t and the return on capital r_t (the latter two of which are being co-determined in the same period t).

It turns out, however, that not every level of interest rates is reachable through open-market operations. First, $j_t \geq 0$ (the zero lower bound), otherwise nobody would be willing to leave the AM holding bonds. Second, $j_t \leq 1/q_t - 1$, otherwise nobody would be willing to leave the AM holding money. In between these bounds – which is the interesting case – Equation (17) holds and active interest rate policy is feasible.

In steady state, the primary market price on bonds equals the secondary market price, so j is the interest rate in both markets: $j = 1/s^B - 1 = 1/p^B - 1$. It is bounded by:

$$0 \leq j \leq i$$

This is sensible. The return on liquid bonds cannot be negative (otherwise, nobody would want to hold them), or exceed the Fisher rate (which is the hypothetical rate of return on an illiquid bond; otherwise, nobody would want to hold money).

3 The economy in the long run

In steady state, all real variables must be constant. Ongoing open-market operations are not necessary once the ratio of bonds to money is at the steady state desired by the government, hence $N_t = 0$. Nominal variables (s_t^K, P_t, M_t, and B_t) must grow at the same rate (we define $\mu \equiv M_{t+1}/M_t$ in gross terms). The transversality condition requires that $\mu \geq 1/(1+\rho)$ (otherwise there would be an infinite demand for money).

For the detailed derivation of steady-state equilibria, refer to Appendix A.1.3; here, we present a summary. In particular, it turns out to be convenient to summarize the effects of long-run money growth in terms of the Fisher rate $i = \mu(1+\rho) - 1$ rather than the money growth rate μ. With the right combination of money growth and money-to-bonds ratio, any combination of the Fisher rate i and policy rate j that satisfies $0 \leq j \leq i$ is feasible.

To find out how these two instruments affect the long-run economy, we evaluate the Euler equations for bonds, money, and capital in steady state. First, (9) simply reduces to:

$$p^B = s^B = \frac{1}{1+j}$$

Since the fundamental price of a nominal discount bond is $1/(1+i)$, we can also write the bond price as the product of the fundamental price and a *liquidity premium*:

$$p^B = \frac{1}{1+i} \times (1+\ell),$$

where the liquidity premium is thus defined as:

$$\ell \equiv \frac{i-j}{1+j},$$

which must satisfy $0 \leq \ell \leq i$. In what follows, we will see that the most succinct way to write the equilibrium equations is in terms of (i, ℓ), but it should be kept in mind that this is a simple transformation of (i, j).

Second, (8) pins down the PM price of goods (q) in terms of monetary policy:

$$q = \frac{\lambda}{1+i-(1-\lambda)(1+j)} \qquad \Rightarrow \qquad q = \frac{1+\ell}{(1+i)\left(1+\frac{\ell}{\lambda}\right)}$$

This is the place to recall Friedman's (1969) famous argument that money balances are optimized when the marginal cost of holding money is zero, which gave the policy $i = 0$ the name "Friedman rule". At the Friedman rule, $q = 1$, and away from it, $q < 1$; hence, q is a wedge that measures how far away the economy is from the Friedman rule, and we therefore call it the **Friedman wedge**. Notice that for a fixed i, the wedge is brought closest to 1 when $\ell = 0$ – that is, $j = i$, the policy rate being at the maximal level. The reason for this is that bonds represent a way for agents to avoid the inflation tax. When the rate of return on bonds is maximized, the impact of the inflation tax is minimized.[13]

[13]This is subtly related to the main result of Berentsen et al. (2007): in their model, an economy with banks helps agents reallocate money to those who need it the most, which improves the allocation relative to an economy without banks. The reason for this is that in their equilibrium, banks pay the maximum possible interest rates on deposits; in our language, $j = i$. Thus, their result that "the gains in welfare come from the payment of interest on deposits" finds its equivalent in our model in "the Friedman wedge is optimized when the interest rate on bonds is maximized". As we shall see, however, in our model the Friedman wedge is not the only criterion for welfare, which is why $j = i$ is not always an optimal policy.

Third, (10) pins down the marginal revenue product of capital:

$$1 + \rho = (r + 1 - \delta) \cdot \left(1 + \eta \, \frac{i - j}{1 + j}\right) \qquad \Rightarrow \qquad r = \delta + \frac{\rho - \eta \ell}{1 + \eta \ell}$$

Thus, the liquidity premium ℓ is a sufficient statistic for the effect of monetary policy on r.

This is the place to recall Mundell's (1963) and Tobin's (1965) famous argument that inflation should stimulate capital accumulation, since it makes holding money more costly and money and capital are substitutes as stores of value. Since the first-best level of r is $\rho + \delta$, we can define a wedge that measures how far away the return on capital is from its benchmark:

$$\frac{\rho + \delta}{r} = \frac{1 + \eta \ell}{1 - \frac{1-\delta}{\rho+\delta} \, \eta \ell}$$

This wedge describes how a positive liquidity premium on bonds ($\ell > 0$) stimulates the accumulation of capital, therefore we call it the **Mundell-Tobin wedge**. When viewed in terms of the monetary policy instruments (i, j), we see that a high illiquid interest rate i (achieved, for example, through higher inflation expectations) stimulates capital accumulation, but it is a *low* level of the policy rate j that does the same.

Now that we have solved for prices q and r, the production side in the PM (Equation 3) pins down the capital-output ratio. Feeding this back into the production function, we solve for the equilibrium level of output itself:

$$\frac{K}{Y} = \frac{\alpha q}{r} \qquad \Rightarrow \qquad Y = A^{\frac{1}{1-\alpha}} \left(\frac{\alpha q}{r}\right)^{\frac{\alpha}{1-\alpha}}$$

In order to complete the characterization, we divide output by its first-best level (Equation 1) which serves to eliminate the constant. We get:

$$Y = \left(\frac{\rho + \delta}{r} \, q\right)^{\frac{\alpha}{1-\alpha}} \cdot Y^*$$

It is now clear that output will equal its first-best level if and only if the Friedman wedge and the Mundell-Tobin wedge *exactly offset* one another. Substituting the two wedge terms for q and r, we get:

$$\left(\frac{Y}{Y^*}\right)^{\frac{1-\alpha}{\alpha}} \; = \; \Omega(i, \ell) \; \equiv \; \frac{1 + \ell}{(1 + i)\left(1 + \frac{\ell}{\lambda}\right)} \cdot \frac{1 + \eta \ell}{1 - \frac{1-\delta}{\rho+\delta} \, \eta \ell} \tag{18}$$

We call Ω the **monetary wedge**. Its direction is defined such that a higher value of the wedge causes higher investment and output.[14] The effect of monetary policy on macroeconomic

[14]Consumption is increasing in Ω iff $\Omega < (\rho + \delta)/\delta$. Beyond this, higher Ω would push capital accumulation

aggregates, in steady state, is fully described by the monetary wedge:

$$Y = A^{\frac{1}{1-\alpha}} \left(\frac{\Omega\alpha}{\rho + \delta} \right)^{\frac{\alpha}{1-\alpha}} \qquad K = \frac{\Omega\alpha}{\rho+\delta} \cdot Y \qquad C = \frac{\rho + (1-\Omega\alpha)\delta}{\rho+\delta} \cdot Y \tag{19}$$

3.1 Optimal long-run policy

Comparing these equations with the social planner's solution from Section 2.2, it is clear that the economy is at its optimum if and only if $\Omega = 1$. When $\Omega > 1$, then output is inefficiently large. When $\Omega < 1$, then output is inefficiently small.

But what values does Ω take? Consider first the extreme case where the policy rate is at its upper bound: $j = i$ and therefore $\ell = 0$ (the liquidity premium is zero, indicating that neither bonds nor capital are priced for their liquidity services). In that case:

$$\Omega_{[\ell=0]}(i) = \frac{1}{1+i}$$

The only policy that achieves $\Omega = 1$ with a zero liquidity premium is the Friedman rule, $i = 0$. Note, for later, the derivative of $\log(\Omega_{[\ell=0]})$ at $i = 0$:

$$\left. \frac{d \log \left(\Omega_{[\ell=0]}\right)}{di} \right|_{i=0} = -1$$

Consider next the other extreme, the zero lower bound where $j = 0$ and therefore $\ell = i$ (bonds are so scarce that the liquidity premium is maximal):

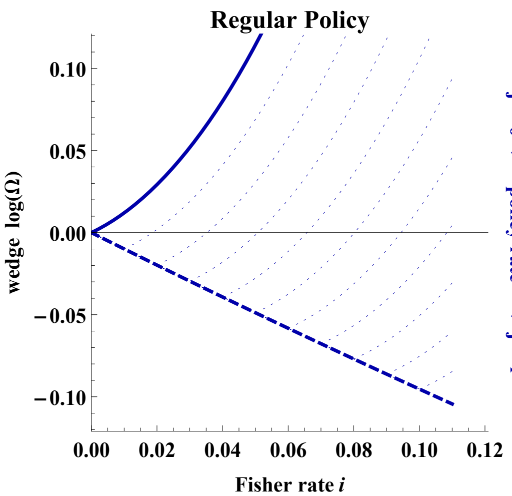
$$\Omega_{[j=0]}(i) = \frac{1}{1+\frac{i}{\lambda}} \cdot \frac{1+\eta i}{1 - \frac{1-\delta}{\rho+\delta}\eta i}$$

This term may be greater or smaller than 1, although it also satisfies $\Omega_{[j=0]}(0) = 1$ (there is no distortion at the Friedman rule). The term blows up when $i \to (\rho+\delta)/[(1-\delta)\eta]$. Hence, if inflation is high enough, the zero lower bound on the bond interest rate cannot be attained; the demand for bonds will hit zero at a positive interest rate, and an interest rate below this level cannot be part of an equilibrium.

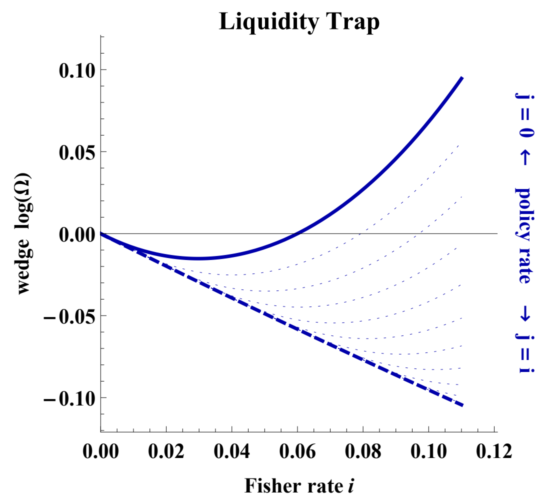
For low inflation, we can classify equilibria into three cases – illustrated in Figure 2 – depending on the derivative of $\log\left(\Omega_{[j=0]}\right)$ at $i = 0$:[15]

beyond its Golden Rule level, and steady-state consumption would decrease.

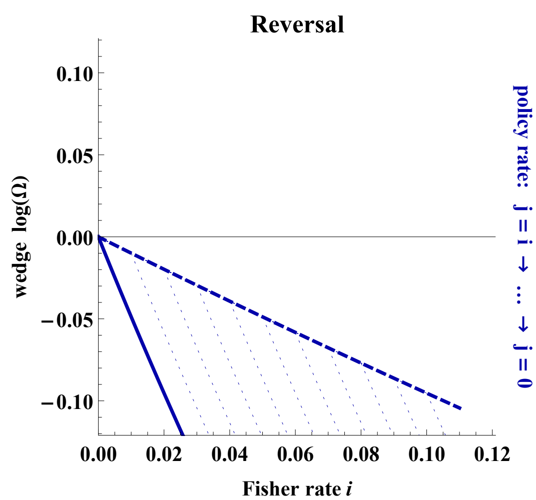
[15] *Proof*. First, solve for $\Omega_{[j=0]} = 1$. This gives a quadratic equation with two generic solutions: $i = 0$ and one more, call it i_1. Recall that $\Omega_{[j=0]}$ blows up as i increases (unless $(1-\delta)\eta = 0$, in which case there is only the trivial solution $i = 0$). Therefore, either $i_1 < 0$, in which case $\Omega_{[j=0]} > 1 > \Omega_{[\ell=0]}$ for all $i > 0$, or $i_1 > 0$, in which case $\Omega_{[j=0]} < 1$ for low enough i. Second, solve for $\Omega_{[j=0]} = \Omega_{[\ell=0]}$. Again, there exist two generic solutions: $i = 0$ and one more, call it i_2. Because $\Omega_{[j=0]}$ blows up but $\Omega_{[\ell=0]}$ does not, $i_2 < 0$ implies $\Omega_{[j=0]} > \Omega_{[\ell=0]}$ for all $i > 0$. Conversely, $i_2 > 0$ implies that $\Omega_{[j=0]} < \Omega_{[\ell=0]} < 1$ for low enough i. Third, since $i = 0$ is always



[a] High η



[b] Intermediate η



[c] Low η (capital is illiquid)

Figure 2: The cone of policy options, for the three cases described in the text.
The continuous line is $\log(\Omega_{[j=0]})$ (policy rate is zero), and the dashed line is $\log(\Omega_{[\ell=0]})$ (policy rate is maximal). Dotted lines indicate policy rates of 1% to 10%.
Positive values of $\log(\Omega)$ represent overinvestment and overproduction, negative values vice versa, and any point on the horizontal axis represents first-best.
Maintained parameters: $\rho = 0.03$, $\delta = 0.1$, $\lambda = 0.2$. Varying parameter: $\eta = \{0, 0.5, 0.75\}$.

$$\left.\frac{d\log\left(\Omega_{[j=0]}\right)}{di}\right|_{i=0} = -\frac{1}{\lambda} + \frac{\rho+1}{\rho+\delta}\,\eta \tag{20}$$

High η: "regular policy". Suppose the term (20) is zero or positive. Then, $\Omega_{[j=0]} > 1$ for all $i > 0$, which means that there exists an interior policy interest rate $j \in (0, i)$ that achieves the first-best $\Omega = 1$. Hence, while $j \geq 0$ is a lower bound on the policy rate, it is not a "trap"; the policy maker should never want $j = 0$ in the first place.

Intermediate η: "liquidity trap". Suppose the term (20) is within $[-1, 0)$. Then, for low inflation rates we have $\Omega_{[j=0]} \in \left(\Omega_{[\ell=0]}, 1\right)$. This means that the lower bound $j \geq 0$ is a binding constraint on policy: the policy maker would like to achieve $\Omega = 1$, but cannot do so by setting j alone. Instead, there are *two* ways to escape the trap: reduce inflation to the Friedman rule, or increase it sufficiently so that $\Omega_{[j=0]} \geq 1$ again, which is always possible if $(1-\delta)\eta > 0$.

We analyze this case (and justify its name) in detail in Section 4.4 below.

Low η: "reversal". Suppose the term (20) is less than (-1) (which is always the case for $\eta \to 0$). In that case, for low enough inflation rates we have $\Omega_{[j=0]} < \Omega_{[\ell=0]}$, a reversal of the previous ranking. To achieve the first-best, i must be reduced to zero. However, conditional on a fixed $i > 0$, the second-best policy is to ramp up the policy rate j to its *maximum*, $j \to i$. The Mundell-Tobin effect is so weak that it is dominated by the Friedman effect, and if the policy maker cannot enact the Friedman rule for some reason, then the second-best policy is to maximize bond interest rates in order to give a boost to money demand.

The "reversal" case is similar to many New Monetarist models where a higher bond supply (or bond interest rate) always increases output, and usually increases welfare too, whereas higher inflation does the opposite (e.g., Williamson, 2012). The LAMMA replicates this result at the limit $\eta \to 0$, where capital is illiquid. But the conclusions – and policy prescriptions – of the models radically diverge when η is high enough: in that case, higher interest rates (for a given inflation rate) reduce investment and output, whereas a higher inflation rate (for given interest rates) increases investment and output, precisely because high η makes capital a liquid asset, and as such a substitute for bonds and money.

4 Lessons from the long-run model

This section includes four subsections, discussing the long-run relationship between inflation and interest rates (4.1), the long-run return on capital (4.2), the optimality of the Fried-

a solution of $\Omega_{[j=0]} = \Omega_{[\ell=0]} = 1$, we can classify the ranking of $(\Omega_{[j=0]}, \Omega_{[\ell=0]})$ for low inflation by comparing their log derivatives at $i = 0$.

man rule (4.3), and the liquidity trap (4.4). They can be read independently.

4.1 Inflation and interest rates

As we discussed earlier, one can describe steady-state equilibrium in terms of quantity instruments (money supply M and bond supply B, and their growth rate μ), or rate instruments (policy rate j and Fisher rate i; or, more realistically, the policy rate j and the inflation target $i - \rho$). Evaluating the AM clearing equation (17) at steady-state values, we get the equation that links the two approaches:

$$\frac{B}{M} = \frac{1-\lambda}{\lambda} \cdot (1+j) \; - \; \frac{\alpha\eta}{1 - \beta(1-\delta)\left(1 - \eta + \eta\frac{1+i}{1+j}\right)}$$

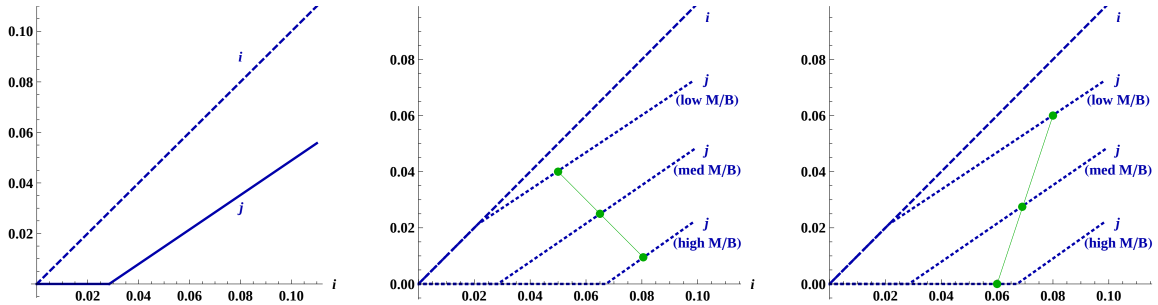
This is to say that, everything else equal, the bond-to-money ratio B/M is a monotonically increasing function of the bond interest rate j and a decreasing function of the Fisher rate i. However, the equation also says that *holding the bond-to-money ratio fixed* instead, j is a monotonically increasing function of i; see panel [a] of Figure 3 for an illustration.

This fact holds true in all monetary models where bonds are imperfectly liquid, and it may tempt one to think that i and j must be positively linked in reality, too. Indeed, almost all of monetary theory treats i as the main instrument of monetary policy, and the fact that higher inflation causes higher yields on liquid bonds in this way is taken as evidence that distinguishing between i and j is not of first-order importance.[16]

Here, we propose a very different approach, treating the interest rates i and j as *distinct policy instruments of equal standing*. Crucially, they can be set independently, and therefore need to obey no particular empirical relationship at all.

The main instrument for monetary policy is j, the yield on liquid bonds, and j is implemented via open-market operations that alter the bond-to-money ratio. The Fisher rate i is determined by the expected path of inflation and output growth; thus, depending on the prevailing shocks in the economy and the monetary policy regime, the empirical correlation between i and j may be positive, negative, or zero. We illustrate these possibilities in Figure 3. As empirical work by Herrenbrueck (2019b) shows, the three panels correspond well to the three main eras of the post-war U.S. monetary experience: the gold-standard/monetarist era where interest rates increased more slowly than inflation ([a]), the Volcker disinflation ([b]), and the Taylor rule era ([c]) where policy rates were made to respond aggressively to inflation and output growth (both of which are components of i).

[16]Even authors of this paper used to conflate the two: "in steady state the growth rate of the money supply pins down the inflation rate, and through... the Fisher equation, this pins down the nominal interest rate; it does not matter if policy controls money, inflation or the interest rate, since any one determines the other two." (Geromichalos et al., 2007)



[a] Standard view in monetary theory: j is a monotonic function of i. Comparative statics with respect to the bond supply may be considered, but are of secondary interest.

[b] If tighter monetary policy decreases the money stock and expected money growth at the same time, then i and j may be *negatively* correlated.

[c] If high expected inflation causes a tightening of monetary policy (for example via a Taylor rule), then i and j may be *positively* correlated.

Figure 3: Comparative statics of j, the yield on liquid bonds, with respect to the Fisher rate i and the bond-to-money ratio B/M, in steady state. Parameters: $\rho = 0.03$, $\delta = 0.1$, $\alpha = 0.36$, $\eta = 0.35$, $\lambda = 0.45$ maintained. Panel [a]: $B/M = 0.15$. Panels [b] and [c]: $B/M \in \{0.03, 0.15, 0.25\}$.

4.2 The long-run return on capital assets

In the previous section, we have discussed the implications of the LAMMA for the long-run return on bonds. The implications for the return on capital are just as interesting. We evaluate Equation (10) in steady state, subtract depreciation (to obtain the net return), add inflation (to make the return nominal), and linearize to obtain the approximate long-run return on capital:

$$\tilde{r} \;\equiv\; r - \delta + \pi \;\approx\; \eta j + (1-\eta) i$$

Furthermore, using the approximate long-run Fisher equation (16) to substitute the components of i (inflation π, real GDP growth γ times the inverse elasticity of intertemporal substitution σ, and the time discount rate ρ), we can write it as:

$$\tilde{r} \;\approx\; \eta j + (1-\eta)\pi + (1-\eta)\sigma\gamma + \text{constant} \tag{21}$$

Of course, this equation is a linear approximation and thus neglects second-order terms. This is fine if the risk premium is just a constant, but if it depends on interest rates, inflation, or GDP growth, the equation would be misspecified. Still, as it is, the equation implies four testable hypotheses:

(H1) Inflation affects long-run capital asset returns, with a coefficient between 0 and 1, if we control for the policy rate j or a proxy (a short-term, risk-free rate) and GDP growth;[17]

[17] Theory implies that the real interest rate on a not-perfectly-illiquid asset should be negatively affected by

(H2) Interest rates affect long-run capital asset returns, with a coefficient between 0 and 1, if we control for inflation and real GDP growth (the components of i);

(H3) The sum of the two coefficients is 1;

(H4) More liquid types of capital have a lower coefficient on inflation/growth and a higher coefficient on interest rates.

In order to test these hypotheses, we used the Jordà-Schularick-Taylor Macrohistory Database (Jordà, Knoll, Kuvshinov, Schularick, and Taylor, 2019); specifically, the time series on annual equity returns (the portion of capital traded on stock markets), housing returns, and returns on the sum of these two categories (quantity-weighted) as outcome variables, and nominal GDP growth, inflation, and short-term interest rates as independent variables. Since the hypotheses all relate to the long run, we followed Jordà et al. (2019) and constructed decade-by-decade averages of the series, resulting in a panel of 16 developed countries across up to 15 decades.[18] We first ran regressions with real GDP growth and inflation entered separately; in all cases, their coefficients are not statistically distinguishable. Thus, in what follows, we work with logarithmic utility ($\sigma = 1$), which implies that the long-run Fisher rate equals the nominal GDP growth rate up to a constant.

Decade-averaged returns on:	(1) Equity		(2) Housing		(3) Total of both	
	coefficients	*coeff. sum*	*coefficients*	*coeff. sum*	*coefficients*	*coeff. sum*
NGDP growth (i)	0.33		0.52		0.48	
	(0.08)	1.45	(0.07)	1.14	(0.06)	1.21
Short-term interest rates (j)	1.12	(0.10)	0.62	(0.09)	0.73	(0.09)
	(0.13)		(0.12)		(0.12)	
R^2	0.31		0.52		0.55	
Observations	224		196		191	
Countries	16		16		16	

Notes: Random-effects regressions of asset returns (decade-averaged) on nominal GDP growth and short-term interest rates (also decade-averaged), in a panel of 16 developed countries for up to 15 decades (since 1870, if available). Cluster-robust standard errors in parentheses. Fixed-effects and pooled regressions give nearly identical results. Data source: www.macrohistory.net/data (Jordà et al., 2019).

Table 1: Estimates of Equation (21).

Our results are shown in Table 1. Hypothesis 1 is strongly supported: the i-coefficients are fairly precisely estimated, and strongly statistically different from both 0 and 1. The sup-

expected inflation, which has been noted many times: e.g., by Geromichalos et al. (2007) for equity, Geromichalos and Herrenbrueck (2016) for bonds, and Venkateswaran and Wright (2014) for bonds, housing, and capital. The latter paper also reviews the previously existing empirical support for this proposition, which is considerable.

[18] We excluded a single observation, Germany in the 1920s, where inflation averaged 10 billion percent.

port for Hypothesis 2 is nearly as strong; all j-coefficients are strongly statistically different from 0, but in case of equity the point estimate exceeds 1 (although the 95%-confidence interval does extend below 1). Hypothesis 3 supported for housing but not for equity, or the total (the 95%-confidence interval for these does not include 1); the reason for this might be the fact that the linearized equation neglects the risk premium, and previous research has shown that indeed the risk premium on capital assets tends to be positively related to policy rates (Bernanke, Gertler, and Gilchrist, 1996; Rocheteau et al., 2018), so it is likely that that coefficient is overestimated. Finally, Hypothesis 4 is supported if we accept the common belief that housing is less liquid than equity: the difference between the i-coefficients is marginally statistically significant ($p = .073$), the difference between the j-coefficients is strongly statistically significant ($p = .005$), and the signs are as expected.

4.3 Should we run the Friedman rule?

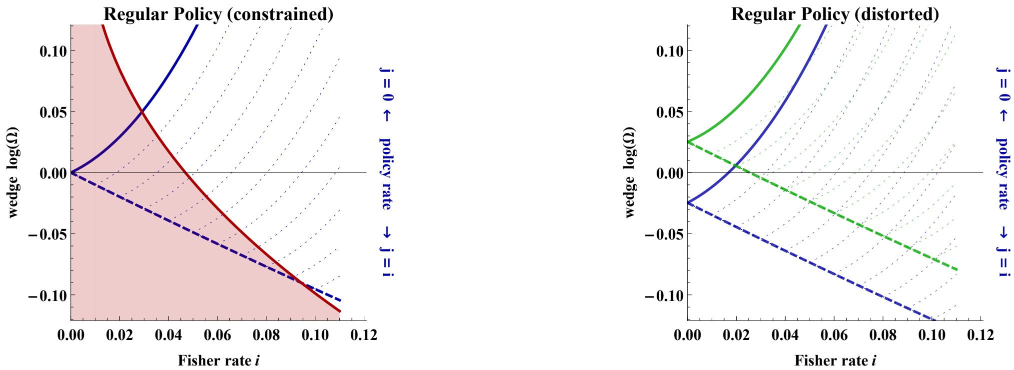
One of the classic questions in monetary economics is the optimal rate of inflation. In the vast majority of models where agents have any reason to hold money (from money-in-the-utility-function all the way to the most modern treatments of frictions), higher inflation induces agents to economize on holding money balances, and thereby receive less of whatever it is money gives them in the particular model. This is the basis for Friedman's rule: set equal the private marginal cost of holding money to the social cost of creating it, which in most models is zero.[19]

However, the Friedman rule may not be feasible, because shrinking the money supply requires retiring existing money, which must be collected through taxes. In the real world, taxes are usually distortionary and hard to collect. To make our point in the simplest possible way, we assume for the moment that lump-sum taxes are not allowed but lump-sum transfers are (following Andolfatto, 2013, and Gomis-Porqueras and Waller, 2017).

Setting $T \geq 0$ in the government's budget constraint does two things. One, it makes the Friedman rule inaccessible. Two, any interest payments on government bonds need to be financed with seigniorage revenue, so zero inflation implies $B = 0$. Depending on other parameters, $B = 0$ may imply that bonds are scarce in the secondary market and thus $j = 0$. In order to achieve the first-best, more bonds must be issued so as to make interest rates rise, which requires further inflation. The counterintuitive implication is that positive inflation is required in order to make "tight" monetary policy (in the sense of high interest rates) possible in the first place. We illustrate this result in Panel [a] of Figure 4.

A common theme in discussions of the optimality of the Friedman rule is whether there are additional distortions in the economy (other than the frictions that make money essen-

[19] A literal reading of Friedman's rule actually prescribes positive inflation if inside money can do some things that outside money cannot do, and the social cost of creating inside money is not zero (such as in some models of banking; e.g., Dong, Huangfu, Sun, and Zhou, 2017).



[a] Shaded area is only feasible with taxes
⇒ high interest rates require inflation

[b] Every optimal policy involves $i > 0$,
whether distortion is positive or negative

Figure 4: The cone of policy options, considering feasibility and distortions.
Parameters: $\rho = 0.03$, $\delta = 0.1$, $\lambda = 0.2$, $\eta = 0.75$.

tial) – search, bargaining, market power, or externalities in general – and whether these are likely to cause overproduction and overinvestment, or the opposite. In models where the distortion causes overproduction, the optimal inflation rate tends to be positive, but in models where the distortion causes underproduction, the Friedman rule is second-best and any deviations from it are especially costly.[20] Formally extending the model to cover the various possibilities is beyond the scope of the present paper, but as a first pass suppose that there is an additional wedge which multiplies the monetary wedge Ω in Equations (19), and thus pushes the economy towards overinvestment or underinvestment (without affecting any other margins). To keep the focus on the monetary mechanism, we are deliberately agnostic as to what this distortion may actually be, and focus on its consequences. In traditional monetary theory – nested in our model as the "reversal" case where capital is illiquid – positive inflation is optimal when the distortion is positive, and the Friedman rule is second-best (and deviations are especially costly) when the distortion is negative. Since positive inflation has a slight benefit in one case but a big cost in the other case, monetary theory has traditionally promoted the Friedman rule as the best real-world monetary policy.

However, the version of the LAMMA where capital is liquid enough yields a very different conclusion, as Panel [b] of Figure 4 illustrates. Positive inflation is just as likely to stimulate investment as it is to hurt it (depending also on the stance of interest rate policy). Furthermore, the optimal policy mix (i, j) involves $i > 0$ *both* in the case of a positive or a negative distortion; in the former case, combined with high interest rates, in the latter case, combined with low interest rates.

[20]Examples in the first category include Head and Kumar (2005) and Herrenbrueck (2017), and the latter category is exemplified by Lagos and Wright (2005) and Aruoba et al. (2011). Both cases are covered in Rocheteau and Wright (2005).

4.4 The liquidity trap

The term "liquidity trap" is widely thrown about but rarely defined, and as a result, there are many models of such a trap in the literature, and they are not all that closely related. The term itself was adapted from Robertson (1940) – "liquidity [...] is a trap for savings" – but the concept was introduced by Keynes (1936): "almost everyone prefers cash to holding a debt which yields so low a rate of interest". During the late 20th century, the concept fell into disregard until it was revived by Krugman (1998) using a New-Keynesian model. In that model, zero interest rates can be a trap because prices are sticky; the economy "wants" either lower prices today, or higher prices tomorrow, but when sticky prices constrain the former and an inflation target constrains the latter, full employment cannot be achieved. Later, Williamson (2012) was the first to talk about the liquidity trap in a model which was explicit about the frictions that made the economy monetary, or defined exactly how the central bank could set interest rates. However, in that model it was no longer clear what was so bad about zero interest rates, or what made them a "trap" for policy: the zero interest rate is an indication that liquid bonds are scarce, so the right thing to do for a fiscal authority is to create more, and the right thing to do for a monetary authority is to not buy them up.

Because there are so many competing uses of the term and no clear definition, for our purposes we define a **liquidity trap** as follows:

(i) The policy interest rate is at a lower bound (which could be zero or something else), but interest rates on less liquid assets are not (the economy is not at the Friedman rule)

(ii) Output is below its optimal level

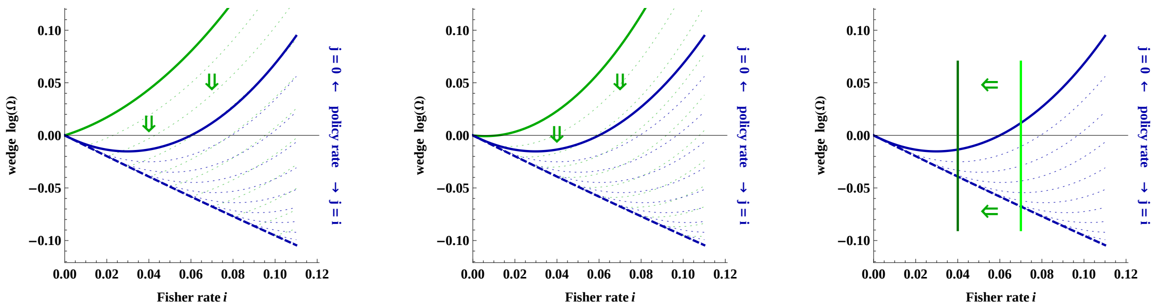
(iii) Raising interest rates would make things worse (hence, "trap")

When is the LAMMA economy in a liquidity trap? The first criterion is $j = 0 < i$ (or, equivalently, $\ell = i > 0$; the liquidity premium is maximal as the policy rate is minimal). As shown in Section 3.1, the rest depends on $\Omega(i, i)$, the value of the monetary wedge when the policy interest rate is zero. If $\Omega(i, i) < 1$, then the second criterion is satisfied, and if furthermore $\Omega(i, 0) < \Omega(i, i) < 1$, then the third criterion is satisfied, too.

But what could push the economy into the liquidity trap? The inequalities can be translated as saying that the term (20) is negative but not too negative, and that i is not too large. This suggests three possible culprits, as illustrated in Figure 5:

(a) A fall in the frequency of liquidity needs ($\lambda\downarrow$). Relatively more households want to buy assets in the secondary asset market, and fewer want to sell them. The equilibrium prices of bonds and capital rise, until their returns hit the respective lower bounds.

(b) A fall in the tradability of capital ($\eta\downarrow$). Capital becomes harder to sell in the secondary asset market; hence, both bonds and the remaining saleable part of capital become more valuable, until their returns hit the respective lower bounds.



[a] Fall in λ (from 0.3 to 0.2) [b] Fall in η (from 0.6 to 0.5) [c] Fall in i (from 7% to 4%)

Figure 5: How an economy can "fall" into a liquidity trap: three plausible candidates. Parameters: $\rho = 0.03$, $\delta = 0.1$, $\eta = 0.5$, $\lambda = 0.2$ (except where indicated otherwise).

(c) A fall in the Fisher interest rate ($i \downarrow$), which could be due to increasing patience or a fall in expected inflation or growth. Either way, keeping $j = 0$ constant, the liquidity premium $\ell = i$ falls. As this premium compensates capital investors for the inflation tax – which falls, too – the combined effect may be to increase or reduce welfare, as Panel [b] of Figure 2 shows.

It is worth noting that just being in the liquidity trap does not cause deflation on its own. On the contrary, a liquidity trap is consistent with any inflation rate that preserves the inequality $\Omega(i, 0) < \Omega(i, i) < 1$. Moreover, the price level in the liquidity trap is still governed by a quantity equation (derived in Appendix A.1.2); it is just not the usual one:

$$\underbrace{\lambda \left[\frac{M+B}{P} + (r+1-\delta)\,\eta\,K \right]}_{\text{real PM expenditure}} \quad = \quad \underbrace{q\,Y}_{\text{market value of PM output}} \tag{22}$$

The left-hand side of the equation equals the sum of all real balances held by shoppers in the PM; this is less than the total supply of real balances M/P (which would be on the left-hand side of the quantity equation if interest rates were away from the zero lower bound).

What does this mean for monetary policy? As bonds are traded at a price of 1, open-market operations that swap money for bonds are neutral; they have no effect on any equilibrium variables, unless they happen to increase the bond supply sufficiently to drive up interest rates (something that would *not* be desirable, by definition of the trap). A helicopter drop of money is neutral for real variables, but it will still affect the general price level P.

Are there other policy options in the liquidity trap? Since the trap manifests itself as depressed investment, we conjecture that a variety of fiscal policies targeting investment could be useful (such as a tax credit, or direct spending by the fiscal authority), but to investigate them properly is beyond the scope of this paper. What we know is that there are two *mon-*

etary policy options: run the Friedman rule, or increase expected inflation until the lower bound on the policy rate no longer binds.

The model does make clear that short-term interventions will have short-term effects, whereas the liquidity trap is in principle a long-term phenomenon. Thus – unless the trap is caused by some temporary shock, such as a fall in η during a financial crisis that can be expected to abate over time – escaping a liquidity trap is not a matter of "priming the pump". If the conditions that pushed the economy into the trap (λ, η, etc.) are expected to last, then medium-run forward guidance about interest rates is less likely to be successful than a permanently higher inflation target.

Paradoxes

Since its inception, the liquidity trap has been associated with a **paradox of thrift**. The argument is that a higher desire by agents to save would normally lead to more investment and output; however, this increase in saving requires a well-functioning financial system and an unconstrained interest rate in order to be *translated* into investment. The LAMMA can capture this mechanism: in the liquidity trap, the policy interest rate is zero and therefore the liquidity premium on capital is maximal, but the premium is still too low to stimulate investment to its optimal level.[21]

What would an increase in the "desire to save" do in this case? The term can be given two possible meanings: first, patience increases ($\rho \downarrow$), and second, agents perceive a lack of spending opportunities ($\lambda \downarrow$). A quick look at Equation (18) and Panel [a] of Figure 5 clarifies that a fall in λ always reduces investment and output – not just in the liquidity trap, but whenever policy interest rates are held fixed. A rise in patience, on the other hand, is generally ambiguous, because it is the inverse of asking what happens if expected inflation falls. We already know that with interest rates held at zero, the effect of inflation on welfare could go either way (see Figure 2); the same is true for patience.

Recently, the New Keynesian model of the liquidity trap has been shown to imply two other paradoxes: the **paradox of toil** and the **paradox of flexibility** (Eggertsson, 2011; Eggertsson and Krugman, 2012). The former states that higher potential output (e.g., TFP) reduces output at the zero lower bound, and the latter states that higher price flexibility (less stickiness) does the same. These two paradoxes do not exist in the LAMMA, even in the liquidity trap. First, lower TFP or a lower capital stock always reduce output and welfare; this is consistent with evidence that contractionary supply shocks are contractionary, even at the zero lower bound (Wieland, 2014). Second, the model shows that a liquidity trap can be understood as a phenomenon of monetary and financial frictions which can occur even when prices are perfectly flexible.

[21] However, the LAMMA also clarifies that this reasoning does not always apply. If capital tradability is altogether too low, then we are in the "reversal" region and investment can be increased by *raising* interest rates.

5 The economy in the short run

In order to make the short-run model interesting, we add a few details that we have been abstracting from in the analysis of the long run. First, we introduce shocks to productivity (A_t) and intertemporal preferences (ρ_t). Second, we modify the utility function to include an elastic labor supply margin:

$$U_t(c_t, h_t, g_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \kappa_0 \frac{h_t^{1+\kappa}}{1+\kappa} + g_t,$$

where κ_0 is a normalizing constant and κ is the inverse Frisch elasticity. In the short-run, some aspect of production has to be elastic; since capital is determined one period ahead, a labor margin is needed.

Finally, we need to make more specific assumptions about the monetary policy apparatus. The monetary authority controls either the AM intervention N_t or the AM bond yield j_t – one of these is dictated by the other – and the money growth regime. For interest rates, there are three interesting options we want to study: (I.1) a policy of no AM intervention ($N_t = 0$), letting interest rates adjust endogenously; (I.2) a Taylor-rule policy of making interest rates respond to inflation and output, letting bond purchases adjust endogenously to clear the asset market; (I.3) a policy of fixed interest rates ($j_t = \bar{j}$), which is of course a degenerate form of (I.2), and which also covers the zero-lower-bound situation when $\bar{j} = 0$.

For money growth, there are two options we consider. First (M.1), a stationary money growth policy where the end-of-period money stock equals the beginning-of-period money stock, scaled up by the long-run inflation target: $M_{t+1} = (1 + \bar{\pi})M_t$. In this regime, it is still possible for the middle-of-period money stock that matters for the PM ($M_t + N_t$) to fluctuate, but these fluctuations are reined in by the end of the period. Alternatively (M.2), we consider a policy of letting "bygones be bygones". What gets scaled up at the end of the period is the middle-of-period money stock, and changes in the money stock due to open-market operations are 'folded in' at the end of the period: $M_{t+1} = (1 + \bar{\pi})(M_t + N_t)$.

The PM money stock generally fluctuates due to the shocks hitting the economy in that period as well as the intervention consistent with the prevailing interest rate policy. Thus, the money growth process under assumption (M.2) is a random walk, whereas under (M.1) it is trend-reverting. Arguably, assumption (I.1) together with (M.1) captures Friedman's proposal of strictly controlled money supply growth (implying endogenous interest rates), assumption (I.2) together with (M.2) captures the post-Volcker era of Taylor rules, and (I.3) together with (M.1) captures the zero-lower-bound episodes of recent decades.[22]

Since we are focusing on monetary policy in this paper, we assume the simplest possi-

[22](I.3) also captures the counterfactual world of "what would happen if the central bank just set interest rates fixed forever?", which is receiving attention from Modern Monetary Theory and associated policy proposals.

THE SHORT-RUN MODEL – VARIABLES

Endogenous Variables

Y_t	Output
C_t	Consumption
H_t	Labor supply
K_t	Capital stock, beginning-of-period
w_t, r_t	Real wage and rental rate of capital
q_t	Friedman wedge, or real marginal cost of production
P_t	General price level ($ price of output in the CM)
$\pi_t \equiv P_t/P_{t-1}-1$	Realized inflation (net)
ℓ_t	Ex-post liquidity premium
j_t	Secondary market bond yield (monetary policy instrument)
M_t	Money stock, beginning-of-period
N_t	New money introduced through bond purchases in the AM

Exogenous Variables

A_t	Technology shock
ρ_t	Intertemporal substitution (natural rate of interest) shock
λ_t	Money demand shock
η_t	Capital liquidity shock
ε_t^j	Policy interest rate shock

Parameters

$\bar{A}$, $\bar{\rho}$, $\bar{\lambda}$, $\bar{\eta}$, $\bar{\varepsilon}^j$	Shock averages; note $\bar{\varepsilon}^j = 0$
σ	Inverse elasticity of intertemporal substitution
κ, κ_0	Labor supply: inverse elasticity, level shifter
α	Capital elasticity of production function
δ	Capital depreciation rate
$\bar{b}$	Ratio of bond supply to money supply
$\bar{\pi}$	Inflation target
τ_a, τ_π, τ_y	Taylor rule coefficients: autoregressive, inflation, output

THE SHORT-RUN MODEL – DYNAMIC EQUATIONS

Euler equation for money: $$\frac{C_t^{-\sigma}}{P_t} = \frac{1}{1+\rho_t}\mathbb{E}_t\left\{\frac{C_{t+1}^{-\sigma}}{P_{t+1}}(1+\ell_{t+1})(1+j_{t+1})\right\}$$

Euler equation for capital: $$C_t^{-\sigma} = \frac{1}{1+\rho_t}\mathbb{E}_t\left\{C_{t+1}^{-\sigma}(r_{t+1}+1-\delta)\left(1+\eta_{t+1}\ell_{t+1}\right)\right\}$$

Definition of the ex-post liquidity premium: $$\ell_t = \lambda_t\left(\frac{1}{(1+j_t)q_t} - 1\right)$$

Labor-leisure choice: $$w_t\, C_t^{-\sigma} = \kappa_0 H_t^{\kappa}$$

Production function: $$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}$$

Capital demand: $$r_t K_t = \alpha\, q_t Y_t$$

Labor demand: $$w_t H_t = (1-\alpha)\, q_t Y_t$$

AM clearing: $$N_t = \lambda_t \cdot \frac{\bar{b}\, M_t + (r_t+1-\delta)\eta_t K_t P_t}{1+j_t} + (\lambda_t - 1)\cdot M_t$$

PM clearing: $$q_t Y_t = \frac{M_t + N_t}{P_t}$$

CM clearing: $$C_t + K_{t+1} = Y_t + (1-\delta)K_t$$

Interest rate policy is either passive ($N_t = 0$), or follows a Taylor rule:

$$1 + j_t = (1+j_{t-1})^{\tau_a}\cdot(1+\bar{j})^{1-\tau_a}\cdot\left(\frac{Y_{t-1}}{\bar{Y}}\right)^{(1-\tau_a)\tau_y}\cdot\left(\frac{1+\pi_{t-1}}{1+\bar{\pi}}\right)^{(1-\tau_a)\tau_\pi}\cdot(1+\varepsilon_t^j)$$

Money growth rule:

$$M_{t+1} = \begin{cases} (1+\bar{\pi})\, M_t & \text{stationary (consistent with price level targeting)} \\ (1+\bar{\pi})\,(M_t + N_t) & \text{persistent (``bygones are bygones'')} \end{cases}$$

Occasionally-binding ZLBs on policy rate: $$j_t \geq 0$$

... and liquidity premium: $$\ell_t \geq 0 \;\leftrightarrow\; j_t \leq \frac{1}{q_t} - 1$$

ble fiscal policy apparatus. Government spending G_t is fixed at 0, lump-sum transfers T_t adjust endogenously to satisfy the government's consolidated budget constraint (11), and the government issues bonds in the CM in order to keep the bond-money ratio fixed at $\bar{b}$: $B_{t+1} = \bar{b}M_{t+1}$.[23]

Thus, an equilibrium in our short-run model consists of a private-economy equilibrium as defined in Definition 1, along with the fiscal policy apparatus defined in the previous paragraph, and a monetary policy apparatus consisting of one of (I.1)-(I.3) and either (M.1) or (M.2). For the reader's convenience, we list the full set of model parameters, equilibrium variables, and equilibrium equations separately on pages 32 and 33.

We study five shocks in this section. The first two are standard shocks to 'economic fundamentals': A_t represents shocks to total factor productivity, and ρ_t represents shocks to intertemporal preferences (in the language of New Keynesian macroeconomics, ρ_t is the "natural rate of interest"). Next, we consider shocks to the two main 'new' parameters of our model: λ_t (the fraction of households seeking to shop in a period) and η_t (the fraction of capital that can be sold in the AM). Neither is a 'fundamental' shock in the sense of affecting preferences or technology. They can be thought of as 'financial' shocks since they determine the role the various assets play in the economy; they can also be thought of as 'demand' shocks, since an increase in both λ_t and η_t increases the demand for money in the AM, and thus the demand for goods in the PM (if the supply of money is at all elastic). Finally, we consider a shock to the policy interest rate j_t (for (I.2), the version of the model with active interest rate policy) or money growth (see Appendix A.4).

5.1 Calibration

Table 2 shows the parameters used to calibrate the model, and their targets. Most of these are fairly standard in the literature. The target for the time discount rate is higher (5% annually) than usual; this is justified since our ρ has to account not only for pure time preference, but also trend GDP growth, as our short-run model is linearized around a steady state. (At any rate, a high discount rate has no problem coexisting with a low interest rate because of the existence of the liquidity premium.) The only two parameters for which there is no standard reference are the mean fraction of shoppers, $\bar{\lambda}$, and the mean capital tradability, $\bar{\eta}$.

For $\bar{\lambda}$, we first log-linearized the short-term money demand equation (17). Defining short-term money growth as $(M_t + N_t)/M_t$ (newly available money divided by previously available money), it turns out that the semi-elasticity of money growth with respect to interest rate changes is $(\bar{\lambda} - 1)$ – at least, if we properly control for the other variables in the equation ($M_t, P_t, K_t, B_t, \lambda_t$). Thus, using US data from 1960-2017, we ran a 4-lag VAR regres-

[23]Note that this is the *consolidated government* issuing bonds, which could mean that the fiscal authority issues a fixed amount of bonds, but the central bank maintains a bond balance sheet that grows and shrinks as needed to maintain monetary policy.

$\bar{\rho} = 0.0125$	Mean time discount rate	Pure time preference plus GDP growth (3%+2 % annually) (Herrenbrueck, 2019b)
$\sigma = 1$	Inverse EIS	Log utility; see Section 4.2
$\kappa = \sqrt{0.1}$	Inv. labor supply elasticity	Geom. mean of preferred values in Christiano, Trabandt, and Walentin (2010)
$\kappa_0 = 1$	Labor supply level shifter	Normalized
$\bar{A} = 1$	Mean TFP	Normalized
$\alpha = 0.25$	Capital share	Christiano et al. (2010)
$\delta = 0.025$	Capital depreciation rate	Christiano et al. (2010)
$\bar{\lambda} = 0.22$	Mean fraction of shoppers	Discussed in this section
$\bar{\eta} = 0.5$	Mean capital tradability	Discussed in this section
$\bar{\pi} = 0.005$	Inflation target	2% annually
$\bar{j} = 0.0075$	Mean policy rate	3% annually (implies: $\bar{\ell} = 4\%$ annually)
$\tau_a = 0.75$ $\tau_\pi = 1.5$ $\tau_y = 0.125$	Taylor rule coefficients (for policy I.2)	Smets and Wouters (2007)

Table 2: Calibrated parameters (quarterly) and their targets.

sion with the 3-month T-bill rate (standing in for j_t), the M1 money stock (standing in for M_t), the price level and the capital stock (as controls), and real GDP (as a control for shocks to λ_t), with lags of the policy rate and money stock also serving as controls for the bond-money ratio B_t/M_t. Interest rates were entered as first differences, and the other variables were entered as log first differences. For the full sample, the on-impact semielasticity (of Δj_{t-1} on $\Delta \log M_t$) is -0.67, suggesting $\bar{\lambda} = 0.33$. For the restricted sample 1980-2017 (the period of active interest rate policy), the semielasticity is -0.89, suggesting $\bar{\lambda} = 0.11$. For our calibration, we use the midpoint of these two estimates (0.22).

For $\bar{\eta}$, we make use of our long-run regression estimates in Section 4.2. Since the coefficient on i should be $(1-\eta)$ and the coefficient on j should be η, we can use the regression results to obtain a reasonable range for what η could be in reality. For equity, we conclude that η is no smaller than 0.67 and no bigger than 1 (the theoretical maximum). For housing, we conclude that η is between 0.48 and 0.62. Jordà et al. (2019) suggest that housing represents about half of an economy's capital stock and equity represents a further sixth. Since the remaining part of the capital stock – privately held businesses – is likely to be much less liquid than equity and housing (after all, by definition these are assets that are not traded on liquid markets), we conclude that the "overall liquidity of the capital stock" is probably in the range of 0.4 to 0.6, thus we calibrate $\bar{\eta}$ to the midpoint of 0.5.

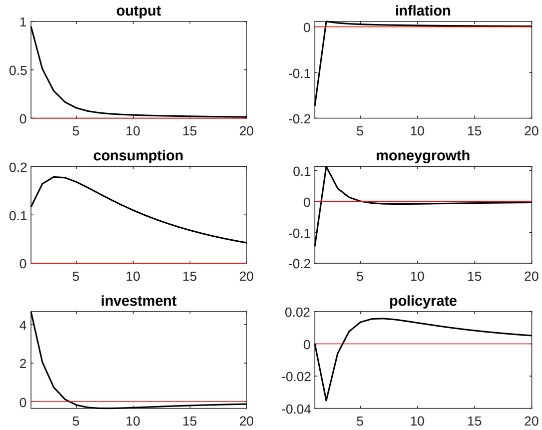
5.2 Effects of shocks

Next, we simulate the short-run response of the economy to our five shocks: total factor productivity, intertemporal preferences, the aggregate fraction of shoppers, the fraction of capital that can be traded in the AM, and the policy interest rate. We assume for this exercise that monetary policy follows the paradigm of 1980-2008 and 2014-2019: interest rates are set according to a Taylor rule (assumption I.2), and money growth is endogenous and persistent (assumption M.2). We show the impulse responses to these shocks in Figure 6.

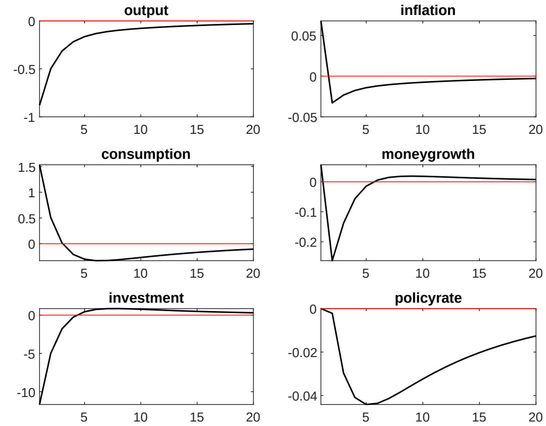
The effects of the TFP shock are quite standard. Output, consumption, and investment increase; investment much more strongly so than consumption. Inflation and money growth (driven by money demand) are low when the shock hits (due to the expanded productive capacity), then catch up. Interest rates respond to output and inflation (with a 1-period lag, as per the Taylor rule), but as the coefficient on inflation is much bigger than the coefficient on output, they mainly track the inverse of inflation; falling at first, then rising.

The effects of the shock to intertemporal preferences – an increase in the "natural rate of interest" – are also fairly standard, at least within the realm of flexible-price models. The increased desire to consume in the present causes households to substitute investment for consumption; the fall in investment leads workers to anticipate lower labor productivity in the future, hence output falls as well. Inflation and money growth increase on impact, but revert in subsequent periods. Policy interest rates fall in order to counteract low output and inflation. It is worth noting that in New Keynesian models (e.g., Smets and Wouters, 2007, who interpret this shock as a "risk premium shock" without giving it a rigorous structural interpretation), such a shock can cause positive comovements of output, investment, and consumption, and is considered an important contributor to business cycle dynamics (Barsky, Justiniano, and Melosi, 2014; Christiano, Eichenbaum, and Trabandt, 2015). However, even in these models this only happens if prices are sticky enough, and investment adjustment costs are high enough.

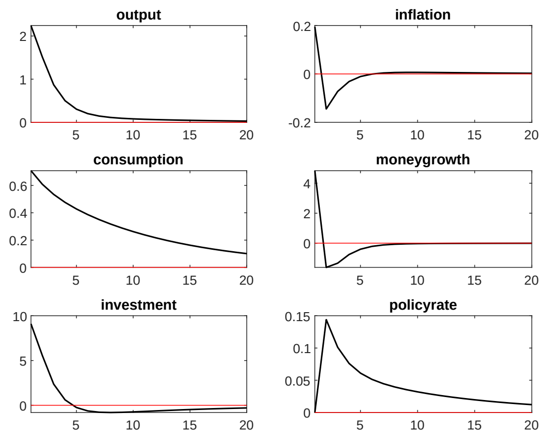
In our model, it is the next two shocks – to the fraction of active shoppers, and the fraction of capital that can be sold in the AM – whose impulse responses look most similar to those of the "risk premium shock" in New Keynesian models. Output, consumption, and investment all increase after a positive shock. Inflation and money growth react positively on impact before being reined in by higher interest rates. Policy interest rates increase and stay elevated for some time after the shock. The reason for these effects is that both of these shocks cause shoppers to demand more money in the AM (either because there are more shoppers, or because they can liquidate more of their other assets). If the money supply was fixed, that demand would translate into higher interest rates on the spot, and *de*flation (see Figure A.5 in the Appendix). However, since interest rates here are governed by a Taylor rule with a lag, the extra money demand is accommodated by the government via an increase in the money



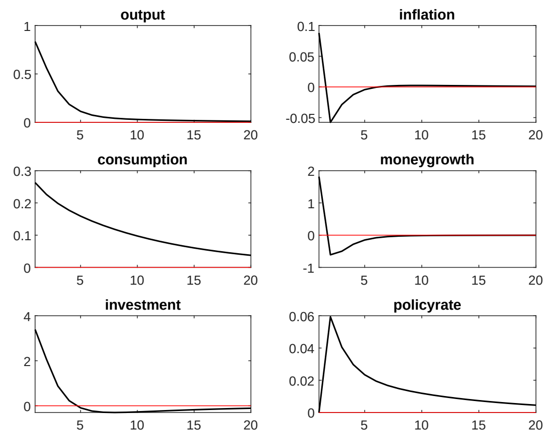
(a) Shock to log TFP ($\varepsilon_t^a = \log(A_t/\bar{A})$)



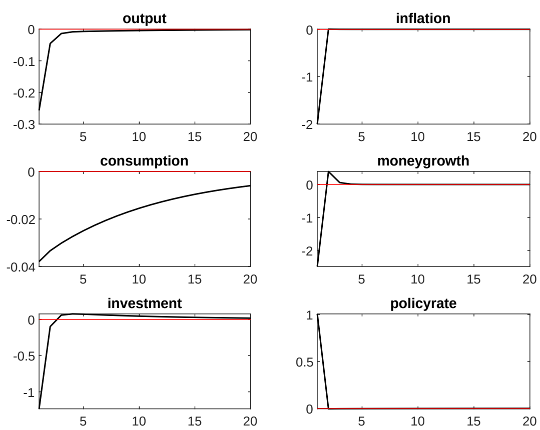
(b) Shock to the natural rate ($\varepsilon_t^n = \rho_t - \bar{\rho}$)



(c) Shock to money demand ($\varepsilon_t^\ell = \lambda_t - \bar{\lambda}$)



(d) Shock to capital tradability ($\varepsilon_t^e = \eta_t - \bar{\eta}$)



(e) Shock to the policy rate ($\varepsilon_t^j = j_t - \bar{j}$)

Notes: Outcome variables are measured in percent (output, consumption, investment, and money growth) or percentage points (inflation and the policy rate). The size of shocks is normalized to 1 percent (TFP) or 1 percentage point (the others). Following standard practice, the first four shocks are persistent with autocorrelation 0.5 but the policy rate shock is not persistent.

Figure 6: Impulse response functions of the dynamic model, with 'standard' policy combination (I.2)/(M.2).

supply, allowing some temporary inflation.

Thus, these shocks arguably answer the challenge from Fisher (2015):

> "Clearly, a deeper foundation for the demand for safe and liquid assets is preferable to the nominally-risk-free-assets-in-utility approach taken here. Nevertheless, the important role for the money demand shock in explaining business cycles suggests that developing a foundation that is amenable to the empirical analysis of aggregate data should be a high priority."

Lastly, we simulate a one-time shock to the policy interest rate. Its effects go in the expected directions: a rise in interest rates causes low inflation, output, consumption, and investment. Two things, however, must be noted. First, the short-term causal effect of interest rates on inflation is unambiguously negative (unlike the long-run effect, which could go either way; see Figure 3) – precisely because high interest rates are implemented by shrinking the money supply in the short term (Equation 17). Second, the direction of the effect of interest rates on real variables matches the long-run model, in that the effect is negative when $\bar{\lambda}\bar{\eta}$ is large enough, but positive when $\bar{\lambda}\bar{\eta}$ is small (see Figure 2, and our discussion in Section 3 of the opposing signs of the Friedman and Mundell-Tobin effects). Our calibration implies that $\bar{\lambda}\bar{\eta}$ is in "large" territory, so that the direction of the effect is as expected: high interest rates cause low investment and output.

It is worth emphasizing that we obtain these comovements in a model where prices are completely flexible. It is likely that introducing some price stickiness in our model would improve its ability to fit the data on inflation in particular; however, our model confirms that sticky prices are *not necessary as a matter of principle* to make monetary shocks, and indeed monetary policy, have plausible effects on real variables.[24]

5.3 Alternative ways to conduct monetary policy

As we explained early on in this section, our model can be used to analyze a large variety of alternative ways to conduct monetary policy, and thus address historical eras other than the period of active interest-setting monetary policy, such as the gold standard, the monetarist experiment, or the zero lower bound era. A deep investigation of monetary history is beyond the scope of the present paper, but in order to illustrate the possibilities, we simulated all combinations of assumptions (I.1)-(I.3) and (M.1)-(M.2) with Dynare. We kept all parameters other than the Taylor rule coefficients as calibrated in Table 2. The results are summarized in Table 3, and impulse responses for two of these cases are shown in Appendix A.4.

We view the LAMMA's ability to address this broad range of possibilities as another benefit of our approach.

[24]Which is, despite decades of debate, still a commonly held belief. Consider Wikipedia: "In macroeconomics, nominal rigidity is necessary to explain how money (and hence monetary policy and inflation) can affect the real economy and why the classical dichotomy breaks down."
(https://en.wikipedia.org/wiki/Nominal_rigidity#Significance_in_macroeconomics)

Policy combinations the LAMMA can be used to analyze		(M.1) Stationary money growth	(M.2) Persistent money growth
(I.1)	Exogenous AM money growth N_t, endogenous interest rates	Friedman's constant money growth proposal. IRFs shown in Figure A.5	Same results as (I.1)/(M.1), plus money is classically neutral
(I.2)	Taylor rule, implemented via AM intervention	Similar results to (I.2)/(M.2)	Post-Volcker era of active interest rate policy. See Section 5.2 and Figure 6
(I.3)	Fixed policy rate $\bar{j}$, implemented via AM intervention	ZLB era, or MMT proposal. IRFs shown in Figure A.6	Indeterminate model; Blanchard-Kahn condition violated

Table 3: Summary of various combinations of interest rate policy and money growth policy.

6 Summary

The LAMMA is a formalization of the following intuitive concepts:

(i) Due to certain frictions, we live in a monetary economy, where many assets are valued – and priced – for their liquidity.

(ii) Financial assets are (generally) imperfect substitutes, and their demand curves (generally) slope down; thus, a central bank that controls the supply of certain assets can "set" interest rates.

(iii) The principal way this is done is via intervention in secondary asset markets where agents rebalance their portfolios in response to short-term liquidity needs.

(iv) The principal channel through which monetary policy affects the economy is the interest rate at which agents save and invest.

As a result, we obtain the following conclusions and lessons for monetary policy:

1. Monetary policy can have real and realistic effects in a tractable model without sticky prices. Such effects are not limited to the short run, but can persist even in steady state.
2. The monetary policy instrument should be modeled as the yield on a highly liquid asset. Thus, (i) it satisfies a *Fisher inequality* rather than the classical Fisher equation; (ii) it can be set independently from expected inflation, via open-market operations; (iii) there is no particular reason for the pass-through from inflation to interest rates to be unity – or even positive.
3. There exists both a real balance effect (inflation causes underproduction) and a Mundell-Tobin effect (inflation causes overproduction). With the right interest rate policy, these

effects offset, thus monetary policy may be able to achieve the first-best long-run outcome at positive inflation rates.

4. Reductions in the policy rate generally increase investment and output – both in the short run and in steady state – but the sign can reverse in certain cases (specifically: if liquidity needs arrive rarely, and if capital assets are hard to trade).
5. There can exist a liquidity trap where the second-best policy is to lower the policy rate to zero, and a first-best policy involves increasing inflation expectations. An economy is likely to be in the trap after a fall in the frequency of liquidity needs ("desire to save rather than spend"), a fall in tradability of capital assets ("shortage of liquid assets"), and a fall in expected inflation ("binding lower bound on real interest rates").

   Conceptually, this liquidity trap is more closely related to that of Keynes (1936) and Hicks (1937) than to the New Keynesian version. For example, there is a paradox of thrift, but improvements in technology still increase output. For another, falling inflation can cause a liquidity trap, but being in a liquidity trap does not (by itself) cause falling inflation.

These results are representative of the topics that the LAMMA can address, but by no means exhaustive. A coequal contribution of the paper is to offer a *framework* facilitating the communication between monetary theory and business cycle macroeconomics. Monetary policy is represented in an empirically relevant way – implemented in secondary asset markets, and treating the policy rate on short-term liquid bonds and the inflation rate as distinct instruments. For this reason, the model is capable of representing not just the current policy regime (active interest rate policy combined with an inflation target and endogenous money growth), but also historical (money growth rule) and counterfactual alternatives (e.g., fixed interest rates). In a simple quantitative exploration, we demonstrate that the empirical implications of the model are plausible in size and sign. Further monetary, financial, and real frictions can be incorporated as needed in future work (e.g., see Herrenbrueck and Strobel, 2020). The potential that such a framework has for quantitative policy analysis is obvious.

Appendix

A.1 Derivations

A.1.1 From value functions to Euler equations

The representative household's value function in the CM subperiod, V^{CM}, is given by Equation (4). Our first task is to confirm that it will be linear. The first-order conditions with respect to the two consumption goods yield $u'(c_t) = P_t/p_t^G$; this does not depend on the household's asset portfolio, only on market prices, therefore every household will consume equal amounts of the c-good. Because $1/p^G$ is the value of marginally relaxing a household's budget constraint by a dollar, and this budget constraint is linear in all state variables, the value function will indeed be linear in all state variables. The envelope conditions are:

$$\partial_m V^{CM} = \partial_b V^{CM} = \frac{1}{(1-\delta)P_t} \cdot \partial_k V^{CM} = \frac{1}{P_t} \cdot \partial_y V^{CM} = \frac{u'(c_t)}{P_t} \tag{A.1}$$

Next, we turn to the household's problem in the AM, where the value functions of asset buyers and sellers are given by Equations (5)-(6). There are three constraints; asset buyers cannot spend more money than they have, asset sellers cannot sell more bonds than they have, and asset sellers can only sell a fraction η_t of their capital. Denote the Lagrange multipliers on the three constraints by $(\theta_t^M, \theta_t^B, \theta_t^K)$, respectively, and shorten notation by writing V_Λ^{PM} for $V^{PM}(\dots, \Lambda)$. Hence, the first-order conditions with respect to supply and demand of assets can be written as follows:

$$\partial_m V_0^{PM} + \theta_t^M = \frac{1}{s_t^B}\, \partial_b V_0^{PM} \qquad\qquad \partial_b V_1^{PM} + \theta_t^B = s_t^B\, \partial_m V_1^{PM} \tag{A.2}$$

$$\partial_m V_0^{PM} + \theta_t^M = \frac{1}{s_t^K}\, \partial_k V_0^{PM} \qquad\qquad \partial_k V_1^{PM} + \theta_t^K = s_t^K\, \partial_m V_1^{PM} \tag{A.3}$$

Combining the two equations on the left, and substituting the envelope conditions for V^{PM} and V^{CM}, we obtain the asset market no-arbitrage equation (7):

$$\begin{aligned}
\frac{\partial_b V_0^{PM}}{s_t^B} = \frac{\partial_k V_0^{PM}}{s_t^K} \quad &\Leftrightarrow \quad \frac{\partial_b V^{CM}}{s_t^B} = \frac{P_t r_t\, \partial_m V^{CM} + \partial_k V^{CM}}{s_t^K} \\
&\Leftrightarrow \quad \frac{\partial_m V^{CM}}{s_t^B} = \frac{P_t(r_t + 1 - \delta)\, \partial_m V^{CM}}{s_t^K} \\
&\Rightarrow \quad \frac{s_t^K}{P_t} = (r_t + 1 - \delta)\, s_t^B
\end{aligned}$$

To solve the portfolio problem in the primary asset market (the CM), we take the first-order conditions of problem (4) with respect to $(m_{t+1}, b_{t+1}, k_{t+1})$. As we have already established that the marginal value of a dollar in the CM equals $u'(c_t)/P_t$, we can write:

$$\begin{aligned}
\frac{u'(c_t)}{P_t} &= \frac{1}{1+\rho}\, \mathbb{E}_t \Big\{ \lambda_{t+1} \partial_m V_{1,t+1}^{AM} + (1-\lambda_{t+1}) \partial_m V_{0,t+1}^{AM} \Big\} \\
\frac{u'(c_t)}{P_t}\, p_t^B &= \frac{1}{1+\rho}\, \mathbb{E}_t \Big\{ \lambda_{t+1} \partial_b V_{1,t+1}^{AM} + (1-\lambda_{t+1}) \partial_b V_{0,t+1}^{AM} \Big\} \\
u'(c_t) &= \frac{1}{1+\rho}\, \mathbb{E}_t \Big\{ \lambda_{t+1} \partial_k V_{1,t+1}^{AM} + (1-\lambda_{t+1}) \partial_k V_{0,t+1}^{AM} \Big\}
\end{aligned}$$

Taking envelope conditions in the AM, and substituting (A.2) and (A.3):

$$
\begin{aligned}
\partial_m V^{AM}_{0,t+1} &= \partial_m V^{PM}_{0,t+1} + \theta^M_{t+1} = \frac{1}{s^B_{t+1}}\,\partial_b V^{PM}_{0,t+1}\\
\partial_m V^{AM}_{1,t+1} &= \partial_m V^{PM}_{1,t+1}\\
\partial_b V^{AM}_{0,t+1} &= \partial_b V^{PM}_{0,t+1}\\
\partial_b V^{AM}_{1,t+1} &= \partial_b V^{PM}_{1,t+1} + \theta^B_{t+1} = s^B_{t+1}\partial_m V^{PM}_{1,t+1}\\
\partial_k V^{AM}_{0,t+1} &= \partial_k V^{PM}_{0,t+1}\\
\partial_k V^{AM}_{1,t+1} &= \partial_k V^{PM}_{1,t+1} + \eta_{t+1}\theta^K_{t+1} = \eta_{t+1} s^K_{t+1}\partial_m V^{PM}_{1,t+1} + (1-\eta_{t+1})\partial_k V^{PM}_{0,t+1}\\
&= \eta_{t+1} P_{t+1}(r_{t+1}+1-\delta)\, s^B_{t+1}\partial_m V^{PM}_{1,t+1} + (1-\eta_{t+1})\partial_k V^{PM}_{0,t+1}
\end{aligned}
$$

Finally, we substitute the AM, PM and CM envelope conditions into the first-order conditions to obtain the Euler equations for money (8), bonds (9), and capital (10).

A.1.2 Asset market equilibrium

Consider the household's problem in the AM, (5)-(6). Suppose first that the constraint on the money spent by a non-shopper household, $\zeta^B_t + \zeta^K_t \le m_t$, is slack. In that case the Lagrange multiplier θ^M_t equals zero; therefore, Equation (A.2) implies $s^B_t = \partial_b V^{PM}_0 / \partial_m V^{PM}_0$, which equals $\partial_b V^{CM}_0 / \partial_m V^{CM}_0$ since a non-shopper household is not active in the PM and proceeds directly to the CM, which therefore implies $s^B_t = 1$ (or $j_t = 0$) due to Equation (A.1).

Next, suppose that the constraint on the money spent by a non-shopper household is binding, but that the constraint on the bonds sold by a shopper household, $\chi_t \le b_t$, is slack. In that case the Lagrange multiplier θ^B_t equals zero; furthermore, substituting Equation (7) back into Equation (A.3) and comparing the result with Equation (A.2) proves that $\theta^K_t = 0$ as well, so the constraint on selling bonds is slack if and only if the constraint on selling capital is slack, too. Additionally, Equation (A.3) now implies $s^B_t = \partial_b V^{PM}_1 / \partial_m V^{PM}_1$. Since shoppers always spend all their money in the PM, their value function V^{PM} evaluated with all constraints binding is:

$$
V^{PM}(m_t, b_t, k_t, 1) = V^{CM}\big(w_t + r_t k_t, b_t, k_t, m_t/(P_t q_t)\big)
$$

Consequently, $s^B_t = \partial_b V^{PM}_1 / \partial_m V^{PM}_1 = \partial_b V^{CM}_1 / \partial_y V^{CM}_1 \cdot P_t q_t = q_t$, using Equation (A.1); or, in terms of the policy rate, $j_t = 1/q_t - 1$.

Finally, suppose that all three constraints bind, so that $\theta^M_t \theta^B_t \theta^K_t > 0$. In that case, $\zeta^B_t + \zeta^K_t = m_t$, $\chi_t = b_t$, and $\xi_t \le \eta_t k_t$. Plug these into Equations (12), substitute aggregate quantities using Equations (13) and (14), and substitute s^K_t using Equation (7) to obtain:

$$
\lambda_t s^B_t \left[B_t + P_t(r_t + 1 - \delta)\eta_t K_t\right] = N_t + (1-\lambda_t) M_t
$$

If the s_t^B implied by this equation is in the interval $[q_t, 1]$, then the equation holds in equilibrium. If the implied s_t^B is not in that interval, then one of the two cases described earlier applies, and the short side of the market is rationed. Additionally, if we substitute $j_t = 1/s_t^B - 1$ and rearrange, then the equation becomes Equation (17), which holds in any equilibrium where $j_t \in [0, 1/q_t - 1]$. Finally, evaluating Equations (8) and (15) in steady state, together they imply $q = \lambda/[1+i-(1-\lambda)(1+j)]$; thus, the inequality $j \leq 1/q - 1$ reduces to, simply, $j \leq i$.

A.1.3 Steady-state equilibria when policy is set in terms of quantities

We evaluate the equilibrium equations from Section 2 in steady state, but with policy set in terms of quantities (M, B). For this section, it turns out to be more convenient to write the solution in terms of the discount factor $\beta = 1/(1+\rho)$ than the time discount rate ρ. Money still grows at gross rate μ, and the transversality condition now requires that $\mu \geq \beta$. (To express the solutions in terms of (i, j), as in Section 3, replace $s^B \mapsto 1/(1+j)$ and $\mu \mapsto \beta(1+i)$; however, this transformation is only valid in Region (B) as defined below.)

The Euler equations thus take the following form:

$$\frac{\mu}{\beta} = \frac{\lambda}{q} + \frac{1-\lambda}{s^B} \tag{A.4}$$

$$p^B \, \frac{\mu}{\beta} = \lambda \frac{s^B}{q} + 1 - \lambda$$

$$\frac{1}{\beta} = (r + 1 - \delta) \cdot \left(1 + \eta \left[\frac{\mu}{\beta} \, s^B - 1\right]\right) \tag{A.5}$$

The first Euler equation represents the demand for money. The left-hand side is the cost of holding wealth in the form of money: inflation times impatience. The right-hand side is the benefit: the ability to buy goods at price q in the PM and then sell them at the higher price 1 in the CM (with probability λ), or the ability to buy bonds in the secondary market at price s^B and collect the full dividend (with probability $1 - \lambda$).

The second Euler equation represents the demand for bonds. We can divide it by the money demand equation to confirm that $p^B = s^B$ in steady state.

The last Euler equation represents the demand for capital. The left-hand side is the cost of storing wealth in the form of capital: impatience. The first term on the right-hand side is the *fundamental* benefit: the ability to collect capital rents in the future. The second term is an additional value of capital: if $\eta > 0$, then capital also provides a *liquidity service*, and if $s^B > \beta/\mu$ – the price of bonds exceeds its own fundamental value – then both bonds and capital are priced for this service.

Suppose we have solved for prices q and r. Then the production side in the PM (Equation (3)) pins down the capital stock and the capital-output ratio:

$$\frac{K}{Y} = \frac{\alpha q}{r}$$

In steady state, aggregate consumption and capital depreciation must add up to output ($Y = C + \delta K$). Putting these together, we can solve for the rest of the real economy:

$$Y = A^{\frac{1}{1-\alpha}} \left(\frac{\alpha q}{r}\right)^{\frac{\alpha}{1-\alpha}}, \qquad K = \frac{\alpha q}{r} \cdot Y, \qquad \text{and} \qquad C = \left(1 - \frac{\alpha \delta q}{r}\right) \cdot Y \tag{A.6}$$

We see that the equilibrium quantities are fully pinned down by the prices q and r, and the Euler equations show that these prices are in turn determined by s^B, the secondary market price of bonds. Thus, everything hinges on conditions in the secondary market, and on the aggregate supplies of bonds and liquid capital relative to money. It turns out that general equilibrium falls into one of three regions: (A) abundant bond supply, which is obtained when B/M is large; (B) an intermediate region; and (C) scarce bond supply, which is obtained when B/M is small. These regions are illustrated in Figure A.1, and their boundaries are derived in detail at the end of this section.

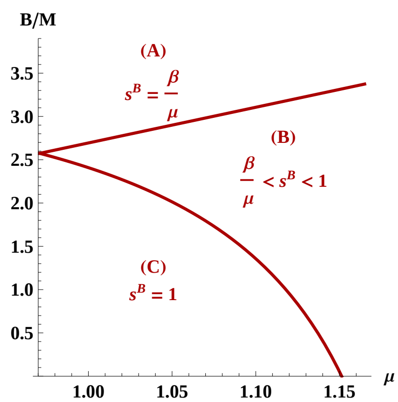


Figure A.1: Regions of equilibrium, in terms of money growth μ and the bond-to-money ratio B/M. Parameters: $\beta = 1/1.03$, $\delta = 0.1$, $\alpha = 0.36$, $\lambda = 0.2$, $\eta = 0.5$.

Region (A): large bond supply

Consider the AM problem described in Equations (5)-(6) and solved in Appendix A.1.1. Because bonds are in large supply, the constraint on selling bonds will not bind. The associated first-order conditions (Equations (A.2)-(A.3)) show that if the constraint on selling bonds does not bind, then neither does the constraint on selling capital. Working through the math, we learn that in steady state:

$$p^B = s^B = q = \frac{\beta}{\mu} \qquad \text{and} \qquad r = \frac{1}{\beta} + \delta - 1$$

Thus, the price of bonds and the return on capital are at their fundamental values, and q (i.e., the ratio between the purchase price of output in the PM and its use value in the CM) is also monotonically decreasing in money growth. Next, we can plug these prices into Equation (A.6), and substitute out the first-best level of output using Equation (1):

$$Y = \left(\frac{\beta}{\mu}\right)^{\frac{\alpha}{1-\alpha}} \cdot Y^*$$

This is the familiar form of the inflation tax. Away from the Friedman rule ($\mu > \beta$), the capital stock and thus output are below their first-best levels for the same reason as in Stockman (1981): income paid to capital owners in period t cannot be used for shopping until period $t+1$, and in the meantime it is subject to the inflation tax.

The flow of expenditure in the PM must equal the value of output. In the AM all money is channeled to the active shoppers, which implies the following quantity equation:

$$\frac{M}{P} = qY = \frac{\beta}{\mu} Y \tag{A.7}$$

Consequently, this region satisfies "Wallace neutrality": changes in the supply of bonds, whether implemented by open-market operations or in any other way, are irrelevant. Money is neutral, too – it affects only the general price level (P) and nothing else – although of course not superneutral. One may think that the liquidity of bonds and/or capital is "irrelevant" here; however, that is not precisely true. The fact that bonds and capital allow agents to purchase money in the AM means that the demand for money is lower than it would otherwise be. This happens not to affect real variables in this region, but it does affect whether we are in this region in the first place. Bonds and capital still provide liquidity services, it is just that they provide them inframarginally.

Region (B): intermediate bond supply

Now, suppose that B/M is smaller than in Region (A), but not too much smaller. In that case, both buyers and sellers of assets in the AM will be constrained, and the market clearing equation in the AM becomes: $[s^B B + \eta\, s^K K] = (1-\lambda)\, M$. After using the no-arbitrage equation (7) to substitute s^K, we obtain:

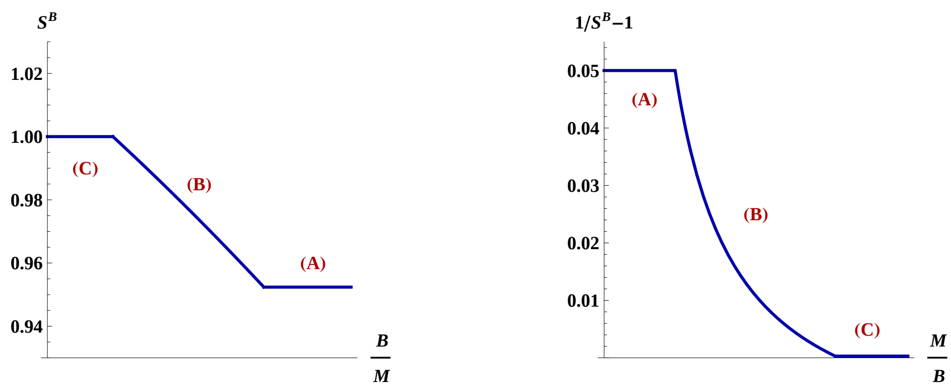
$$\lambda\, s^B \Big(B + P(r+1-\delta)\eta K\Big) = (1-\lambda)\, M$$

Because of CRS in production, capital owners receive a fraction α of total expenditure M; that is, $rK = \alpha M/P$. We use this to substitute K in Equation (17), and the Euler equation to substitute r, and we define the auxiliary term X to get the following expression relating the quantity of bonds with their price:

$$\frac{B}{M} = \frac{1-\lambda}{\lambda} \cdot \frac{1}{s^B} - \frac{\alpha\eta}{1-X}, \qquad \text{where:} \qquad X = \beta(1-\delta)\left[1+\eta\left(\frac{\mu}{\beta}s^B - 1\right)\right] \quad \text{(A.8)}$$

Since $dX/ds^B > 0$, we see that the quantity B/M must be negatively related to the price s^B, and that the equation has a unique implicit solution for s^B in terms of B/M.

Hence, Region (B) is the region of effective monetary policy. An open-market purchase which increases the quantity of money at the expense of bonds will increase the price of bonds and affect the real economy, in the short run through the level of real balances but in the long run through q and r (Equation (A.8), illustrated in Figure A.2). Even a helicopter drop of money which left the quantity of bonds unchanged would work in the same direction and have, unless it was reversed, permanent effects.



[a] Bond demand in terms of bond price   [b] Money demand in terms of the policy rate

Figure A.2: Comparative statics of the bond-money ratio B/M in steady state, interpreted as long-run demand curves for these assets. Panel [b] is the inversion of Panel [a]. Parameters: $\beta = 1/1.03$, $\mu = 1.02$, $\delta = 0.1$, $\alpha = 0.36$, $\eta = 0.35$, $\lambda = 0.45$.

Having thus solved for s^B, we can use the Euler equations (A.4)-(A.5) to find q and r. Differentiating the Euler equations, we see that:

$$\frac{dq}{ds^B} < 0 \qquad \text{and} \qquad \frac{dr}{ds^B} < 0$$

This is intuitive: if bonds are more expensive in the secondary market, then asset buyers will get a worse return on their money. Anticipating this (with probability $1-\lambda$), agents will carry less money in the first place. This goes on until the principal compensation for holding money – the mark-up earned by buying goods in the PM and selling them in the CM, $1/q$ – has increased enough. Furthermore, as bonds are more expensive in both markets, agents will prefer to hold capital as a store of value, leading to an increased accumulation of capital; that is, until the return on capital has fallen enough to make them indifferent again.

Plug these results into Equation (A.6), and we see that the effect of s^B on steady-state output is generally ambiguous. Making bonds scarce (hence, increasing their price) takes away one way for agents to store their wealth and avoid the inflation tax (the Friedman effect). On the other hand, agents will respond by substituting into capital (the Mundell-Tobin effect), which stimulates investment and, ultimately, output and consumption. As we have shown in Section 3, either effect can dominate.

Region (C): low bond supply

In this region, the constraints on selling bonds and capital in the AM do bind, but the constraint on spending money does not. Setting the associated Lagrange multiplier to zero and working through the first-order conditions (see Appendix A.1.1), we learn that $s_t^B = 1$ in any equilibrium. This is intuitive: after the AM has closed, the only benefit the bonds have is to pay out one unit of money in the CM. Hence, in steady state:

$$
\begin{aligned}
p^B &= s^B = 1 \\
q &= \frac{\lambda\beta}{\mu - (1-\lambda)\beta} \\
r &= \frac{1}{\eta\mu + (1-\eta)\beta} + \delta - 1
\end{aligned}
$$

In this region, bond prices are maximal and q and r are minimal. In the AM, agents with a shopping opportunity sell all their bonds and liquid capital ($\chi = B$ and $\xi_t = \eta_t K_t$; bonds are scarce if and only if capital is, too), but the asset buyers are not willing to spend all their money at such high prices. Therefore, the flow of spending in the PM no longer satisfies the standard quantity equation $M/P = qY$. Instead, the general price level is determined by a quantity equation (Equation (22) in the main text) that includes the bond supply.

Money is neutral (except for affecting the general price level, P), but not superneutral: an increase in steady-state money growth will decrease both q and r, in the same way that an increase in bond prices did in Region (B).

The boundary between Regions (A)-(B)

This boundary can be found by computing the trade volume in the AM conditional on being in Region (A). In this Region, we have $\zeta^B + \zeta^K = M$, thus the combined trade volume of bonds and capital is $(1-\lambda)M$. If bonds and capital are to be plentiful in AM trade, then their nominal value (which is $\lambda(s^B B + s^K \eta K)$, to be evaluated at Region-(A) prices) cannot be any smaller. Using the no-arbitrage equation (7), the quantity equation (A.7), and the earlier results for this region, we see that equilibrium is in Region (A) if:

$$\lambda(s^B B + s^K \eta K) \;\geq\; (1-\lambda)M \quad \Rightarrow \quad s^B \left(\frac{B}{M} + P(r+1-\delta)\,\eta\,\frac{K}{M} \right) \;\geq\; \frac{1-\lambda}{\lambda}$$
$$\Rightarrow \quad \frac{B}{M} \;\geq\; \frac{1-\lambda}{\lambda} \cdot \frac{\mu}{\beta} \;-\; \frac{\alpha\eta}{1-\beta+\beta\delta},$$

a threshold which is increasing in the rate of money growth, μ. (It is increasing because a higher inflation rate decreases the bond price, thus making it less likely that a given quantity of bonds will be enough to purchase the available money.) The threshold can also be negative; in that case, the economy will be in Region (A) for any positive quantity of bonds.

The boundary between Regions (B)-(C)

This boundary can be found by plugging the bond price upper bound $s^B = 1$ into Equation (A.8). We see that equilibrium is in Region (C) if:

$$\frac{B}{M} \leq \frac{1-\lambda}{\lambda} - \frac{\alpha\eta}{1-(1-\delta)\left[(1-\eta)\beta + \eta\mu\right]}$$

This threshold is decreasing in the parameters α (capital intensity of production), λ (frequency of liquidity needs), η (tradability of capital), and μ (money growth). It can be negative; in that case, the economy will be in Region (A) or (B) for any positive quantity of bonds. In particular, as μ increases, the second fraction will eventually blow up; this means that *high money growth is incompatible with a zero interest rate on bonds*.

A.2 Extension: trading frictions in the PM

There are two reasons for introducing search frictions explictly into the goods market. One is the fact that we have already assumed that shoppers are anonymous and unable to commit to promises. This fits more naturally with the idea that shoppers meet with only a small number of firms, and trade bilaterally. The second reason is that search frictions give rise to market power (firms receive some of the gains from trade), and to mismatch (some shoppers do not trade). The result of these two things is to make the velocity of money in the goods market endogenous (or, at any rate, more flexible than it was in the main text; see Equation (A.7), which reflected the fact that at least outside of the liquidity trap, every dollar in the economy got spent in the goods market). For future empirical applications, this additional flexibility is likely to be important. The reader may in any case be interested in a version of the LAMMA where firms have market power.

Suppose that there are N firms (where N is large), who are price takers in the factor market; they rent labor and capital at market prices w and r exactly as in the main text. However, they have the ability to post output prices, and shoppers face search frictions: they are subject to a lottery whereby they observe the price of n firms with probability ψ_n (Burdett and Judd, 1983). Draws are independent across shoppers, firms, and time. After observing their set of prices (or none, if $n = 0$), shoppers will choose to spend all their money at the firm with the lowest price. There is no recall of prices seen in previous periods.

Because there is a continuum of consumers and a finite number of firms, the law of large numbers applies and each firm can perfectly forecast demand for its product, conditional on the price it has set. (Hence, this set-up abstracts away from inventory or unemployment concerns.) Now, what is that demand? Each shopper has a certain amount of money to spend, and a constant willingness to pay money for output goods (P_t, as derived in Section 2.3). Since P_t is the same for all shoppers, and independent of their money holdings, all shoppers follow the same optimal strategy: spend all of their money on the firm with the cheapest price available, unless that price exceeds $1/P_t$. In the latter case, spend nothing. Thus, for any firm charging a price below this reservation price, the intensive margin of demand is unit elastic. Based on this intensive margin alone, the best thing for a firm to do would be to set their price equal to the reservation price.

However, there is also the extensive margin to be considered. Suppose that the c.d.f. of posted prices is $F(p)$, and that it has no mass points; then, a firm charging price p' will almost surely sell to $a(F(p'))$ shoppers, where:

$$a(F) = \sum_{n=0}^{\infty} \psi_n n (1-F)^{n-1}$$

Therefore, writing $q_t \equiv A^{-1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1} r_t^{\alpha} w_t^{1-\alpha}$ for the real unit cost of producing one unit of output, and noting that $(M_t + N_t)$ is the amount of money held by the representative shopper in the PM (after possible AM intervention by the monetary authority), nominal profits of a firm with price $p \leq P_t$ are equal to:

$$(M_t + N_t)\left(1 - \frac{q_t P_t}{p}\right) a(F(p))$$

Burdett and Judd (1983) proved that – as long as both $\psi_1 > 0$ and $\psi_n > 0$ for some $n \geq 2$ – the only equilibrium of this price-setting game is endogenous price dispersion, where all firms post the price distribution $F(p)$ as a mixed strategy and make equal profits in expectation. The equilibrium $F(p)$ indeed has no mass points, and some firms do charge the reservation price ($F(p) < 1$ for $p < P_t$). Furthermore, Herrenbrueck (2017) showed that the total amount of output purchased equals:

$$Y_t = \left(\psi_0 \cdot 0 + \psi_1 \cdot 1 + (1 - \psi_0 - \psi_1)\frac{1}{q_t}\right) \cdot \frac{M_t + N_t}{P_t}$$

In words: a fraction ψ_0 of shoppers is mismatched and does not purchase anything (although they still get to hold on to their money). The rest of the solution is surprisingly simple: even though almost all shoppers spend a price in between the efficient price ($q_t P_t$) and their reservation price (P_t), the equilibrium is *as if* a fraction ψ_1 of them spent the reservation price and everyone remaining (who was matched with $n \geq 2$ firms) spent the efficient price, equal to marginal cost.

Thus, the Euler equations representing asset demands change as follows from Equations (8)-(10) (we also allow for the matching parameters ψ_n to change over time). First, the ex-post liquidity premium ℓ_t becomes:

$$\ell_t = \lambda_t s_t^B \left(\psi_{0,t} + \psi_{1,t} + \frac{1 - \psi_{0,t} - \psi_{1,t}}{q_t} \right) - \lambda_t$$

Then:

$$\frac{u'(c_t)}{P_t} = \beta \, \mathbb{E}_t \left\{ \frac{u'(c_{t+1})}{P_{t+1}} \, \frac{1 + \ell_{t+1}}{s_{t+1}^B} \right\}$$

$$\frac{u'(c_t)}{P_t} \, p_t^B = \beta \, \mathbb{E}_t \left\{ \frac{u'(c_{t+1})}{P_{t+1}} \, (1 + \ell_{t+1}) \right\}$$

$$u'(c_t) = \beta \, \mathbb{E}_t \left\{ u'(c_{t+1})(r_{t+1} + 1 - \delta) \, (1 + \eta_{t+1} \ell_{t+1}) \right\}$$

Note that if $\psi_0 = \psi_1 = 0$, i.e., all shoppers see at least two prices, then PM trade is *effectively* competitive and the Euler equations are the same as before. And, more precisely, these Euler equations hold under the assumption that the number of prices a shopper observes (n) is only revealed *after* the AM subperiod has concluded. If this was revealed at the beginning of a period, then shoppers with low n or high observed prices would choose to use their money to buy assets rather than goods in that period (Chen, 2015).

In steady state, and using the two interest rates $1 + j \equiv 1/s^B$ and $1 + i \equiv (1 + j)(1 + \ell)$ again, we obtain the following modified expression for the Friedman wedge:

$$q = \frac{1 - \psi_0 - \psi_1}{\frac{1+i}{1+\ell}\left(1 + \frac{\ell}{\lambda}\right) - \psi_0 - \psi_1}$$

The equations for the Mundell-Tobin wedge and for PM clearing (Equation 3) stay the same. Hence, the resulting monetary wedge follows the same formula as before:

$$\Omega(i, \ell) = q \cdot \frac{\rho + \delta}{r},$$

the only difference being that q now incorporates the matching friction terms ψ_0 and ψ_1. Their effect will be to push the wedge Ω down compared to the main text. First, if $\psi_1 > 0$, then firms have market power and shoppers will give up some surplus. Second, if $\psi_0 > 0$, then there is mismatch, and some shoppers will not be able to make a purchase. However, at the Friedman rule, $i = \ell = 0$ implies $q = 1$, as before. Thus, the effect of matching frictions in the goods market is to *rotate* the cone of policy options downwards around the origin – see Figure A.3 for an illustration. The result of this is that for any given policy interest rate, the inflation tax bites more keenly, and output is lower; equivalently, for any given inflation rate the optimal policy interest rate has to be lower.[25]

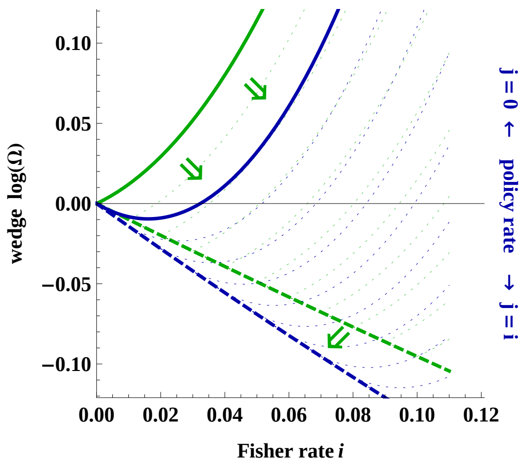


Figure A.3: Menu of policy options without (blue) and with (green) goods market frictions. Maintained parameters: $\rho = 0.03$, $\delta = 0.1$, $\lambda = 0.2$, $\eta = 0.75$. Varying: $(\psi_0 + \psi_1) \in \{0, 0.3\}$.

On the income side, how does the money held by shoppers get distributed after the PM? First, a fraction ψ_0 is unspent by the shoppers, hence they keep it. It can be shown (Herrenbrueck, 2017) that a fraction $(1-q)\psi_1/(1-\psi_0)$ of the remainder – or, equivalently, $(1-q)\psi_1$ of the total – goes to the owners of the firms as profits. The rest gets paid to the owners of factor inputs. Hence, a fraction $\alpha[1-\psi_0-(1-q)\psi_1]$ of the shoppers' money holdings goes to capital owners, and a fraction $(1-\alpha)[1-\psi_0-(1-q)\psi_1]$ goes to workers.

It is now straightforward to take the limit $(\psi_0, \psi_1) \to 0$, meaning that every shopper sees the prices of *at least two* firms. Then, the equations of the frictional model become identical to those of the competitive model in the main text.

Once firms make profits, it may be interesting to model firm equity explicitly. In particular, equity might be considered an indirectly liquid asset that can be sold in the AM, in the same way that capital is (see also Rocheteau and Rodriguez-Lopez, 2014). Other details can be added as desired. For example, there may be firm entry subject to a cost, and entry by more firms may have the effect to improve the matching probabilities by shoppers in the

[25]Unless we are in the "reversal" region of the parameter space, in which case the second-best level of the policy rate, for a given $i > 0$, is maximal – as before.

sense of a FOSD shift in the distribution $\{\psi\}$ (see also Herrenbrueck, 2017).

A.3 Extension: separate fiscal and monetary authorities

In this section, we split the consolidated government into a fiscal authority (in charge of bond issuance, B) and a monetary authority (in charge of the money supply, M), and analyze these authorities' policy options separately. We will not take a deep look into "fiscal policy", which could also include cyclical policy, government spending on a public good, or distortionary taxation, but of course such an analysis could easily be done with the tools developed here.

Suppose that the fiscal authority controls the sequence of bond issues $\{B_t^F\}_{t=1}^{\infty}$ (B_0 is taken as given), and it seeks to finance a sequence of nominal lump-sum transfers $\{T_t\}_{t=0}^{\infty}$ (taxes if negative). The fiscal authority is only active during the CM.

The monetary authority is active during the AM and the CM, and it controls the sequence of money supplies $\{M_t\}_{t=1}^{\infty}$ (again, M_0 is taken as given) and open-market purchases in the AM $\{N_t\}_{t=0}^{\infty}$ (if negative, interpret as sales). From its interventions, the monetary authority may end up holding (or owing) bonds as well; denote its bond holdings at the beginning of period t by B_t^M. With this choice of notation, B_t^F indicates bonds *issued* by the fiscal authority, whereas B_t^M indicates bonds *held* by the monetary authority. The stock of bonds held by the public, at the beginning of period t, will then be $B_t \equiv B_t^F - B_t^M$.

At the end of a period, the monetary authority makes a seigniorage transfer to the fiscal authority, Σ_t. Here, we do not take a stand on whether this transfer can be negative as well as positive, or whether the monetary authority has authority over choosing its level. For example, it may be realistic to assume that the monetary authority has full authority over choosing *positive* levels of Σ_t, but requires the cooperation of the fiscal authority if it wants to cover losses. Alternatively, a monetary authority may have limited power to increase inflation if a more hawkish fiscal authority refuses to increase its spending (Andolfatto, 2015); arguably, this has been the case in the Eurozone in recent years (Bützer, 2017).

Since the fiscal authority issues bonds in the primary market (the CM), where the bond price is p^B, it must obey the following budget constraint, for all $t \geq 0$:

$$p_t^B B_{t+1}^F + \Sigma_t = B_t^F + T_t,$$

along with the no-Ponzi condition that B_t^F / M_t remains bounded. The monetary authority must obey the following budget constraint, for all $t \geq 0$:

$$M_{t+1} - M_t + B_t^M + (1 + j_t) N_t = \Sigma_t + N_t + p_t^B B_{t+1}^M$$

We can interpret this constraint as follows. On the left hand side is the 'revenue' of the monetary authority in period t: newly created money ($M_{t+1} - M_t$) and payments from redemption of the remaining bonds in its portfolio in the CM, after the AM intervention ($B_t^M + (1+j_t)N_t$). On the right hand side are the things this revenue can be spent on: the seigniorage transfer to the fiscal authority (Σ_t), open-market purchases of bonds from the public (N_t), and purchases of newly issued bonds from the fiscal authority ($p_t^B B_{t+1}^M$).

Note that we can add up the budget constraints of the two authorities; the seigniorage term cancels, and after substituting $B_t = B_t^F - B_t^M$ we are left exactly with the budget constraint of the consolidated government (11).

Since any conflict between the authorities' objectives would become manifest in the long run, we proceed by looking at steady states. Assume that the fiscal authority is committed to increasing the supply of nominal bonds at rate μ^B, for a long period of time. (Not, strictly speaking, forever, as the fiscal no-Ponzi condition would be violated if $B/M \to \infty$.) Does that mean that the long-run inflation rate will be μ^B? Not necessarily, because it is still the monetary authority that controls the money stock. But it is now impossible for monetary policy to achieve every point on the 'cone of policy options' derived in Section 3.1 and illustrated in Figure 2. Instead, the monetary authority is left with three options:

(i) Grow the money stock at (gross) rate $\mu^M < \mu^B$ (or shrink it if $\mu^M < 1$). In that case, B/M grows large, and eventually equilibrium must be in Region (A), where the policy rate is maximal, and governed by the rate of money growth: $j = i = \mu^M/\beta - 1$.

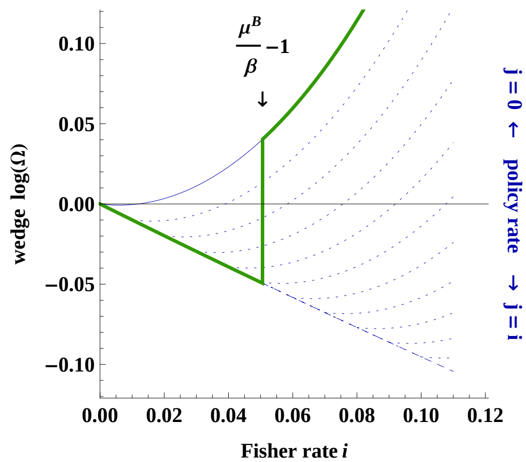
(ii) Grow the money stock at exactly $\mu^M = \mu^B$. In that case, any policy interest rate $j \in [0, i]$ is achievable for the monetary authority.

(iii) Grow the money stock at a faster rate than the supply of bonds: $\mu^M > \mu^B$. In that case, $B/M \to 0$, and eventually equilibrium must be in Region (C), where the policy interest rate is at the zero lower bound: $j = 0$.

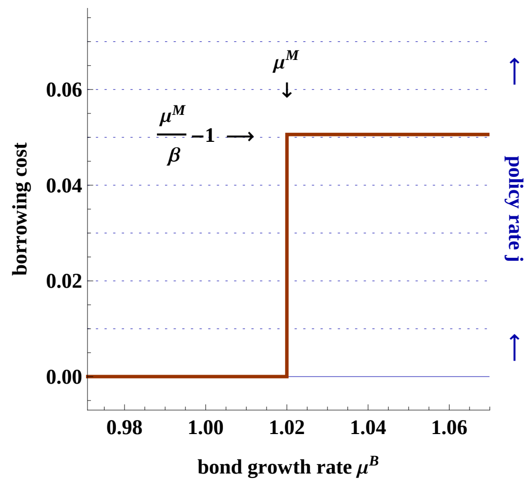
This menu of monetary policy options is illustrated in Panel [a] of Figure A.4. In every case $i = \mu^M/\beta - 1$; that is, the monetary authority controls inflation. However, a benevolent monetary authority seeking to maximize social welfare will have strong incentives to match the money growth rate to the bond supply growth rate, because that is the only way the policy rate can be set to the first-best level. Unless, of course, the monetary authority can implement the Friedman rule; but since this requires $\Sigma_t < 0$, negative seigniorage, they may not be able to do this without cooperation from the fiscal authority.

On the other hand, the fiscal authority may have incentives, too. Even if it is not fully benevolent, it probably still prefers low borrowing costs (policy interest rate j) to high ones. Panel [b] of Figure A.4 illustrates this. Taking the money growth rate μ^M as given, the fiscal authority is left with three options:

(i) Grow the bond supply at (gross) rate $\mu^B > \mu^M$, at least for a while (it cannot be forever due to the no-Ponzi condition). In that case, B/M grows large, and eventually equilibrium must be in Region (A), where the borrowing cost is maximal: $j = i = \mu^M/\beta - 1$.



[a] Monetary policy options (green), constrained by bond growth rate μ^B



[b] Fiscal policy options (red), constrained by money growth rate μ^M

Figure A.4: Menu of long-run options for the monetary and fiscal authorities.

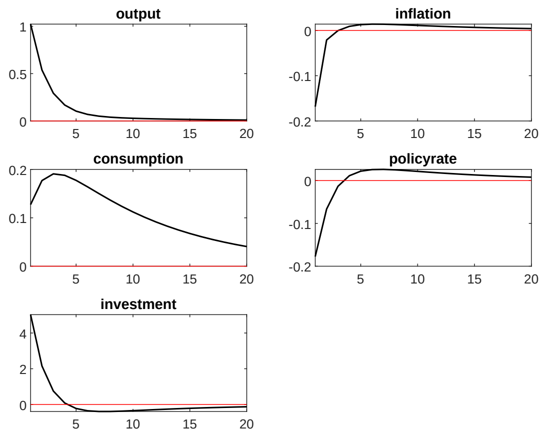
(ii) Grow the bond supply at exactly $\mu^B = \mu^M$. In that case, the monetary authority chooses both i and j, and it is likely to choose $j < i$.

(iii) Grow the bond supply at a slower rate than the money stock: $\mu^B < \mu^M$. In that case, $B/M \to 0$, and eventually equilibrium must be in Region (C), where the borrowing cost is minimal: $j = 0$.

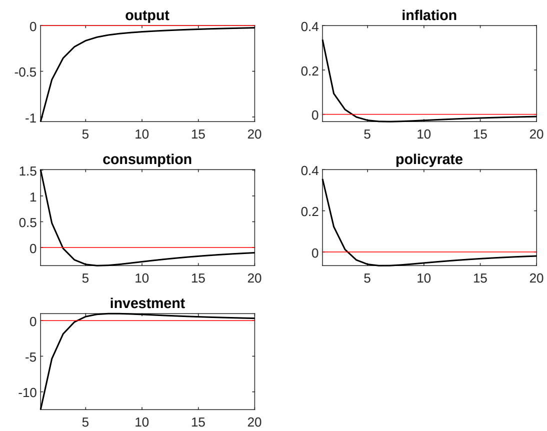
It is beyond the scope of this paper to take a stand on the particular incentives that the two authorities may have, and to analyze this game exhaustively. But we learn a few simple lessons already. First, if the game is non-cooperative, clearly its outcome will hinge on which one of the two authorities has (or is perceived to have) greater commitment power. It stands to reason that the fiscal authority prefers low borrowing costs over high ones, hence it has a strong incentive to grow the bond supply in the long run at approximately the rate of inflation that the monetary authority prefers. However, if the fiscal authority is able to commit to a high rate of bond issuance, then a benevolent monetary authority also has an incentive to give in and accept the long-run inflation rate that the fiscal authority prefers.

A.4 Extensions: short-run policy without a Taylor rule

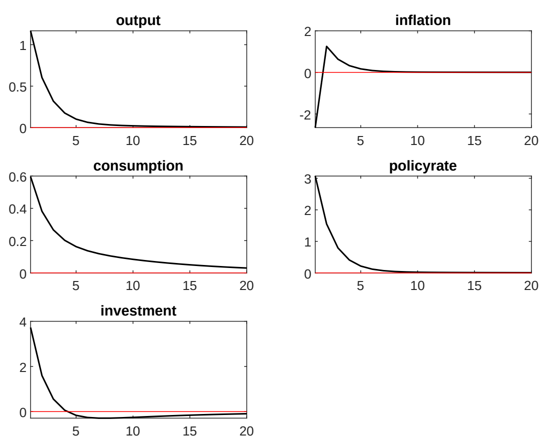
See Figure A.5 for the impulse response functions of the short term model in case of a Friedman-style constant money growth policy, and Figure A.6 for the case of fixed interest rates (such as at the zero lower bound).



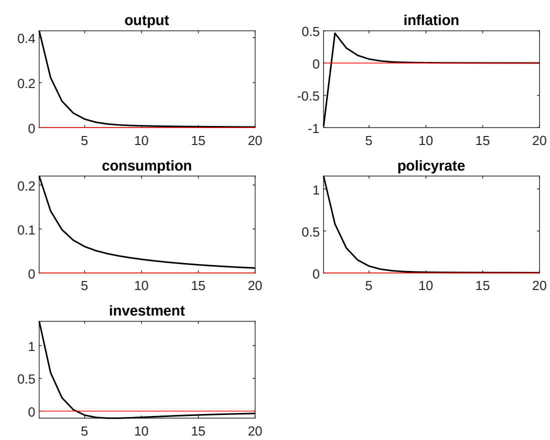
(a) Shock to log TFP ($\varepsilon^a_t = \log(A_t/\bar{A})$)



(b) Shock to the natural rate ($\varepsilon^n_t = \rho_t - \bar{\rho}$)



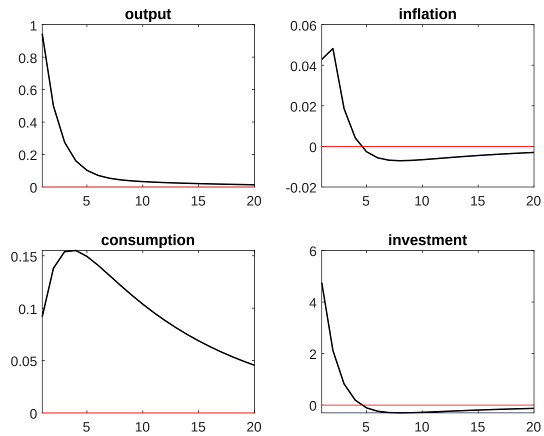
(c) Shock to money demand ($\varepsilon^\ell_t = \lambda_t - \bar{\lambda}$)



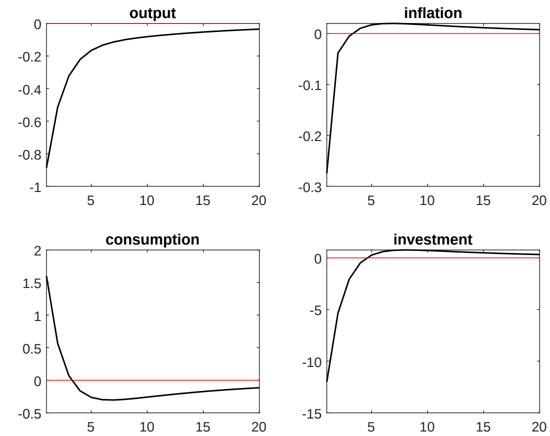
(d) Shock to capital tradability ($\varepsilon^e_t = \eta_t - \bar{\eta}$)

Notes: Unlike in Figure 6 in the main text, money growth is fixed for these simulations, with interest rates endogenously adjusting to clear the AM. (Thus, strictly speaking, j_t is not a "policy" rate but we keep this terminology for consistency with the main text.)
Like in Figure 6, outcome variables are measured in percent (output, consumption, investment) or percentage points (inflation and the policy rate). The size of shocks is normalized to 1 percent (TFP) or 1 percentage point (the others). Following standard practice, all four shocks are persistent with autocorrelation 0.5.

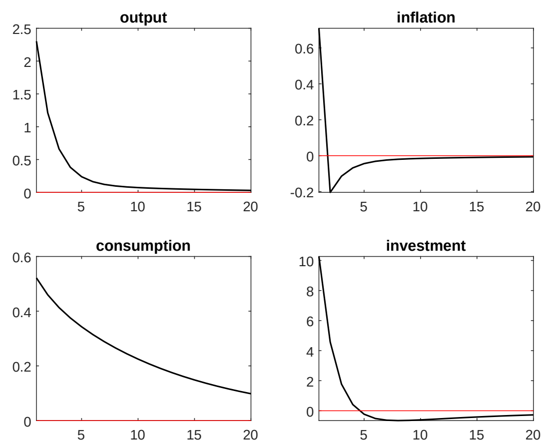
Figure A.5: Impulse response functions of the dynamic model, under monetary policy model (I.1)/(M.1): Friedman-style fixed money growth ($N_t = 0$, $M_{t+1} = (1 + \bar{\pi})M_t$).



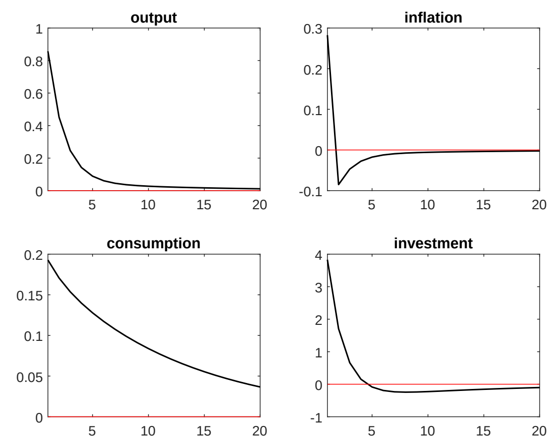
(a) Shock to log TFP ($\varepsilon_t^a = \log(A_t/\bar{A})$)



(b) Shock to the natural rate ($\varepsilon_t^n = \rho_t - \bar{\rho}$)



(c) Shock to money demand ($\varepsilon_t^\ell = \lambda_t - \bar{\lambda}$)



(d) Shock to capital tradability ($\varepsilon_t^e = \eta_t - \bar{\eta}$)

Notes: Unlike in Figure 6 in the main text, both money growth and the policy rate are fixed for these simulations, hence these variables do not have impulse responses.
Like in Figure 6, outcome variables are measured in percent (output, consumption, investment) or percentage points (inflation and the policy rate). The size of shocks is normalized to 1 percent (TFP) or 1 percentage point (the others). Following standard practice, all four shocks are persistent with autocorrelation 0.5.

Figure A.6: Impulse response functions of the dynamic model, under monetary policy model (I.3)/(M.1): ZLB-style fixed interest rates ($j_t = 0$) and exogenous money growth.

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